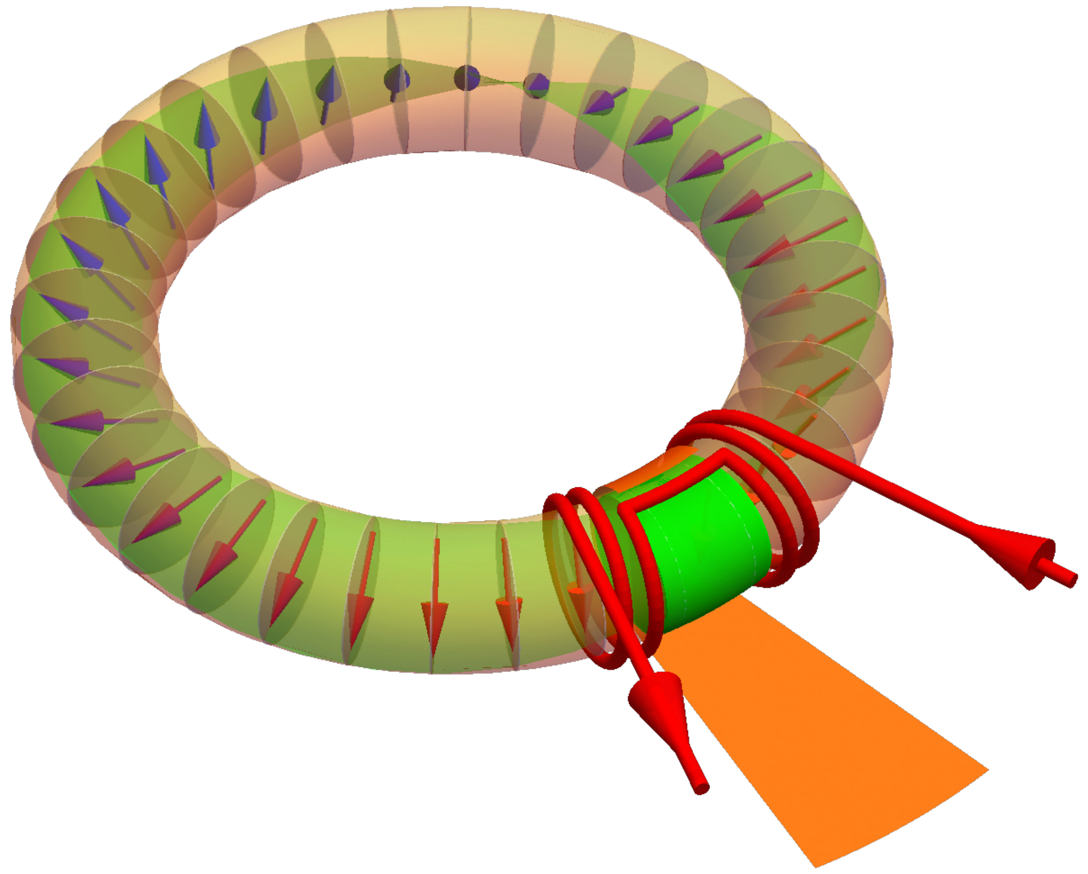
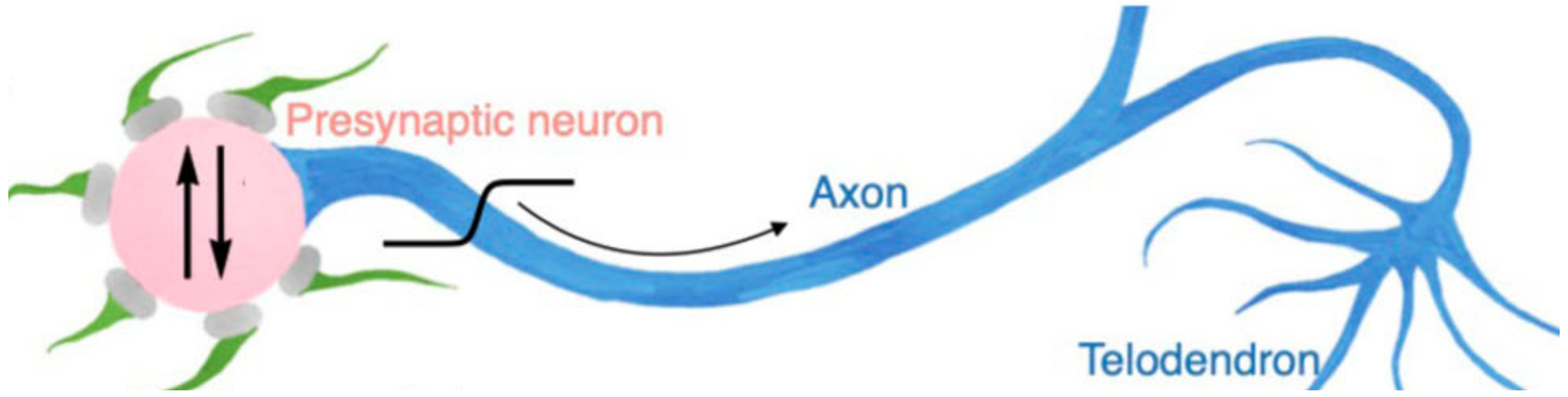
Topological transport: Hydrodynamics and applications

Effective coarse-grained theories and application concepts in *neuromorphics*, *sensing*, and *energy storage*

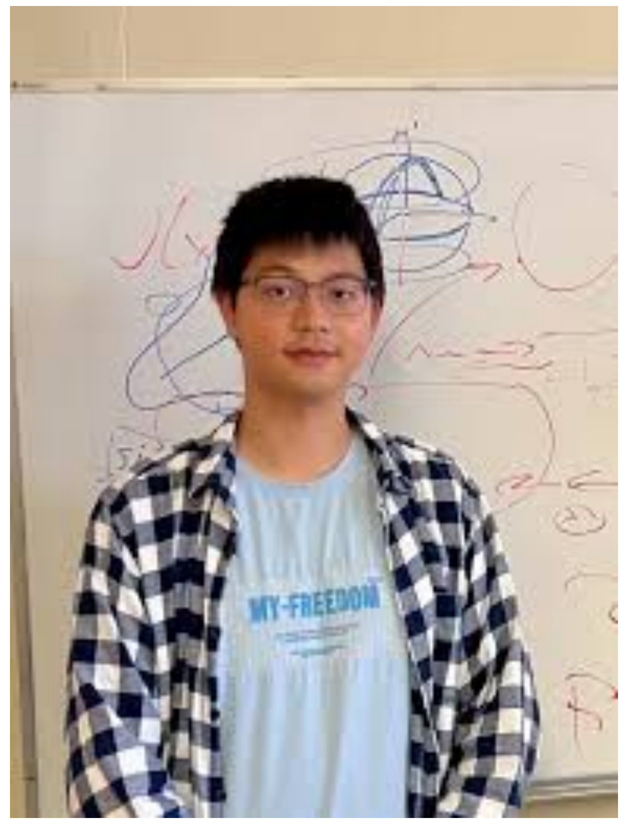




Yaroslav Tserkovnyak

People

Jiang Xiao *(Fudan)*



Yanyan Zhu *(UCLA)*

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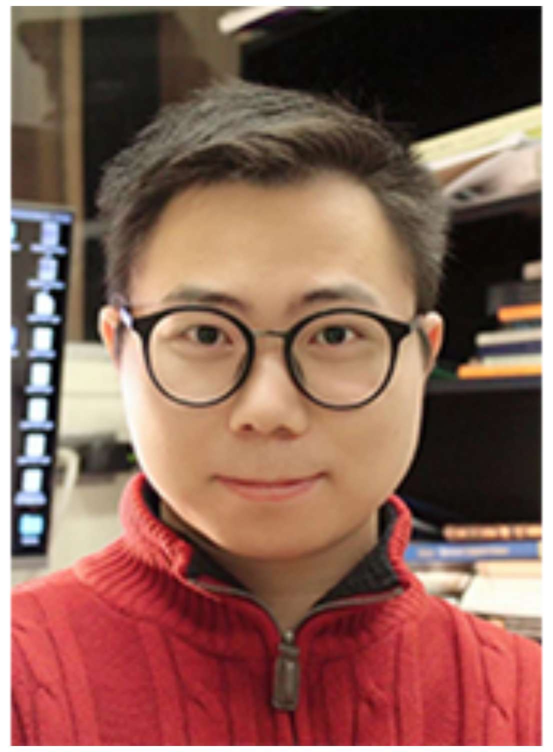
Chau Dao *(Basel)*

Leonid Levitov *(MIT)*



Ji Zou *(KFUPM)*

Suzy Zhang *(Okinawa)*

Eric Kleinherbers *(UCLA)*

(Proof-of-principle) applications validate the concepts

- **Nonchemical energy storage**
  - *Magnetism can provide both (meta)stability and sizable free energy, per atomic bond*

- **Sensing**
  - *Motivated by SQUID, we propose a dual spin-superfluid QUID for electric-field sensing*

- **AI hardware**
  - *Topological degrees of freedom naturally provide nonlinearities, transport/communication, information storage/learning, and interface with standard electronics*

- **Topological conservation laws** in magnetic media and the associated bulk-boundary correspondence engender **energy-storage functionality**

- We exploit **thermoelectricity** as a blueprint for understanding (universal) thermodynamic efficiency of the devices based on **transport of conserved quantities**, such as magnetic vorticity

PHYSICAL REVIEW LETTERS **121,** 127701 (2018)

Editors' Suggestion

Energy Storage via Topological Spin Textures

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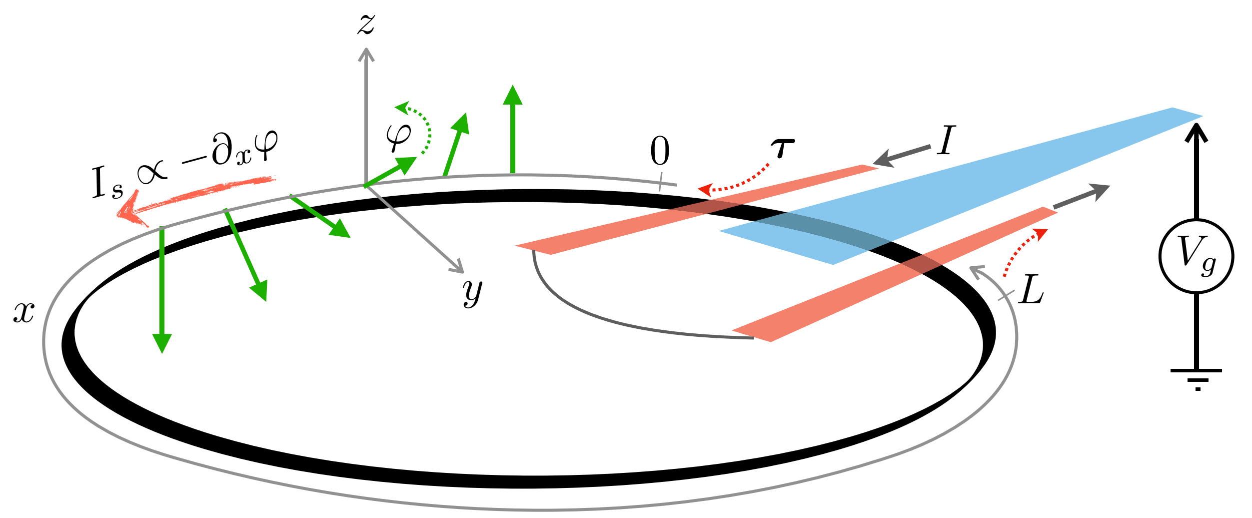
(Received 4 May 2018; revised manuscript received 23 June 2018; published 18 September 2018)

We formulate an energy-storage concept based on the free energy associated with metastable magnetic configurations. Despite the active magnetic region of the battery being electrically insulating, it can sustain effective hydrodynamics of spin textures, whose conservation law is governed by topology. To illustrate the key physics and potential functionality, we focus here on the simplest quasi-one-dimensional case of planar winding of the magnetic order parameter. The energy is stored in the metastable winding number, which can be injected electrically by an appropriately tailored spin torque. Because of the nonvolatility and the endurance of magnetic systems, the injected energy can be stored essentially indefinitely, with the topological charge cycles that do not degrade over time.



PHYSICAL REVIEW B **102**, 140411(R) (2020)

Rapid Communications

Energy storage in magnetic textures driven by vorticity flow

Dalton Jones, Ji Zou, Shu Zhang, and Yaroslav Tserkovnyak
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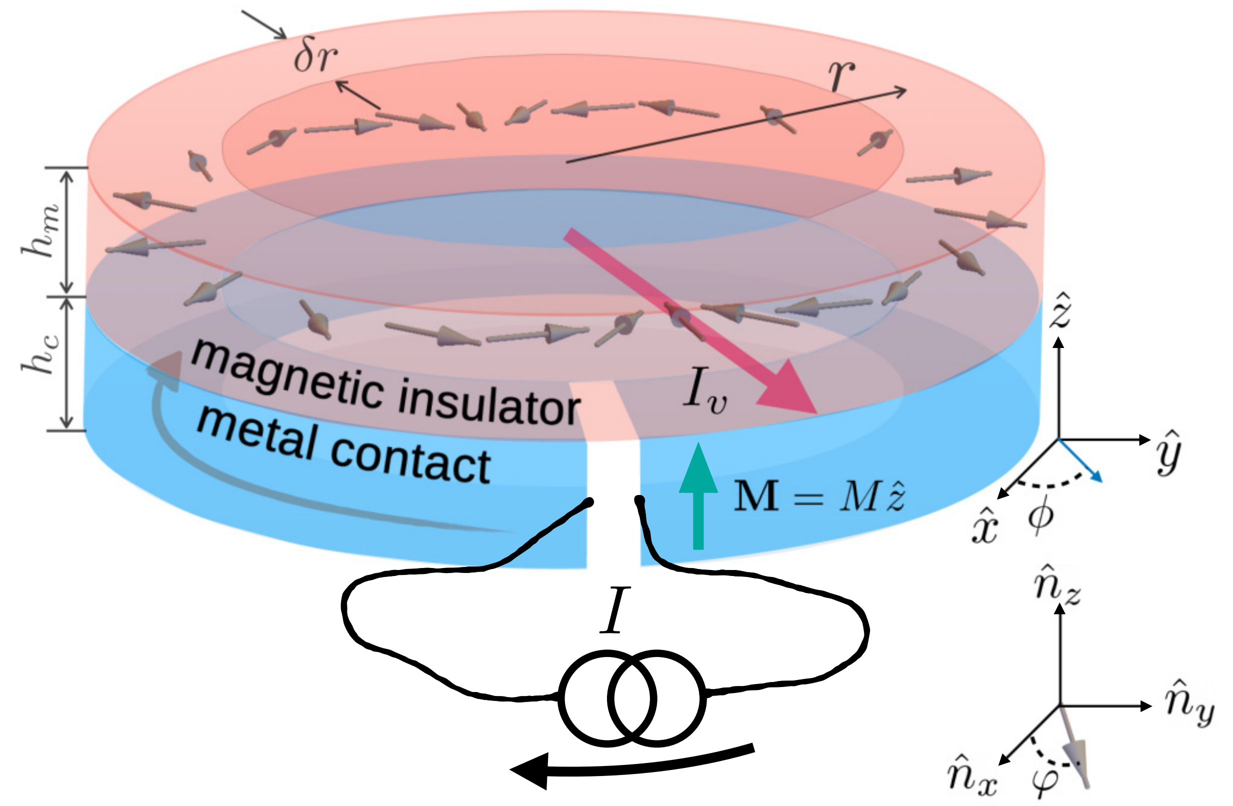
(Received 30 March 2020; revised 15 July 2020; accepted 23 September 2020; published 28 October 2020)

An experimentally feasible energy-storage concept is formulated based on vorticity (hydro)dynamics within an easy-plane insulating magnet. The free energy associated with the magnetic winding texture is built up in a circular easy-plane magnetic structure by injecting a vorticity flow in the radial direction. The latter is accomplished by electrically induced spin-transfer torque, which pumps energy into the magnetic system in proportion to the vortex flux. The resultant magnetic metastable state with a finite winding number can be maintained indefinitely because the process of its relaxation via phase slips is exponentially suppressed when the temperature is brought well below the Curie temperature. We characterize the vorticity-current interaction underlying the energy-loading mechanism through its contribution to the effective electric inductance in the rf response. Our proposal may open an avenue for naturally powering spintronic circuits and nontraditional magnet-based neuromorphic networks.



PHYSICAL REVIEW B **111**, L100401 (2025)

Letter

Topological transport of vorticity on curved magnetic membranes

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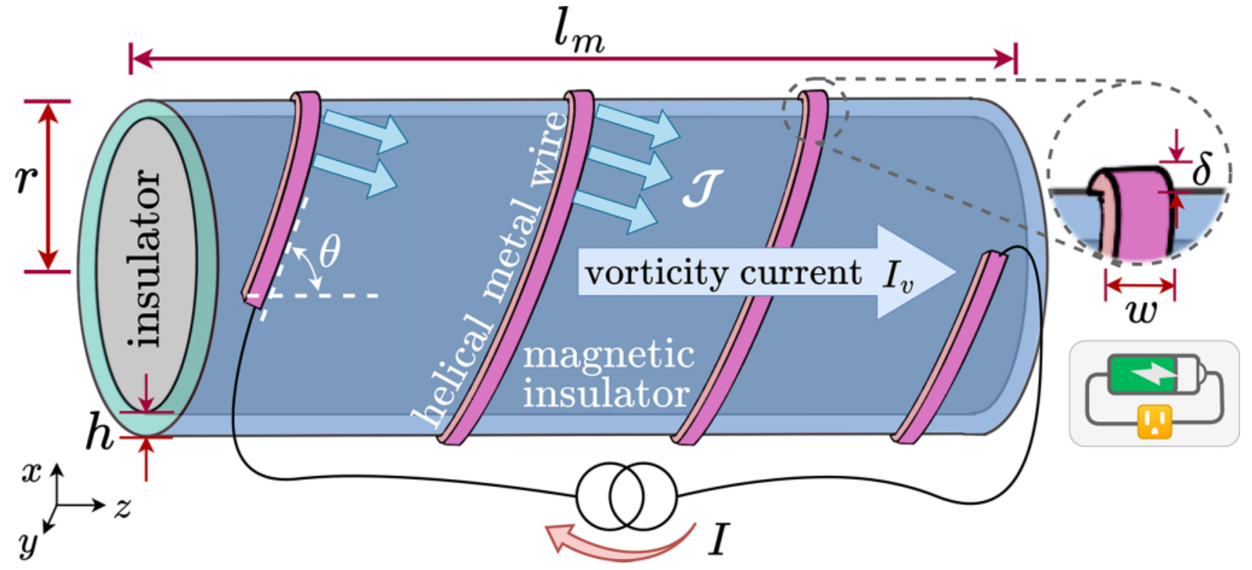
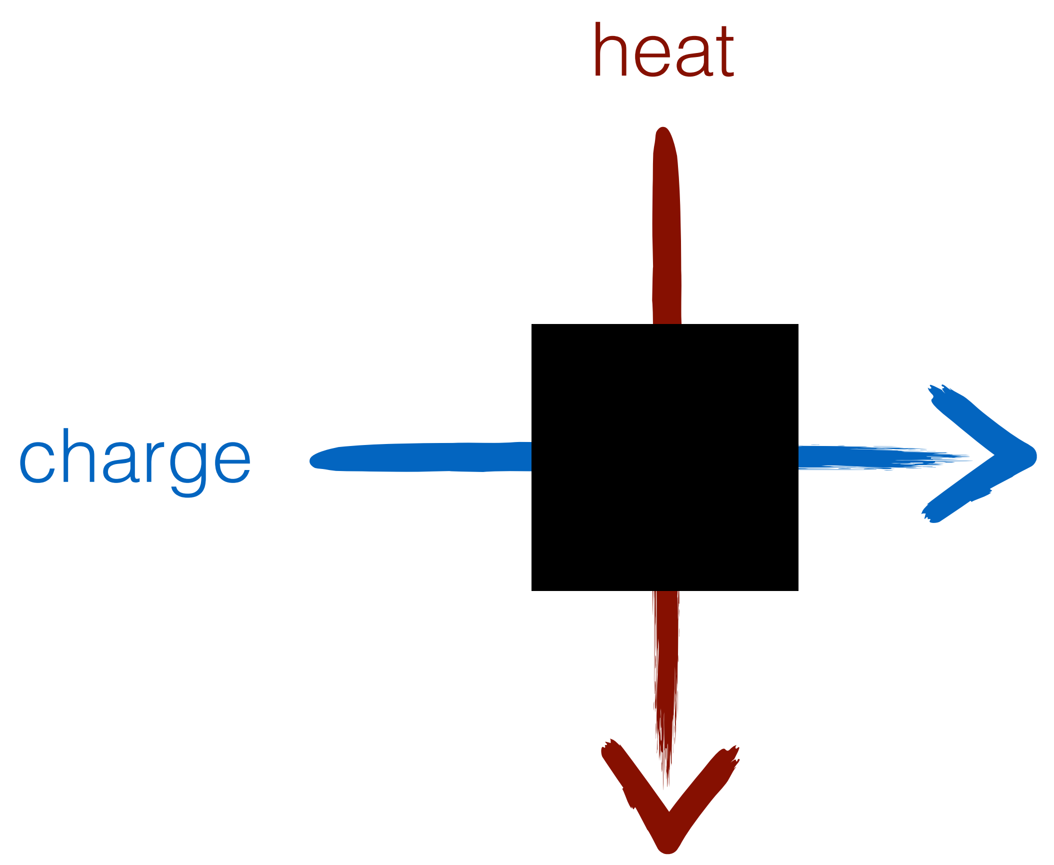
(Received 2 November 2023; revised 21 October 2024; accepted 27 January 2025; published 3 March 2025)

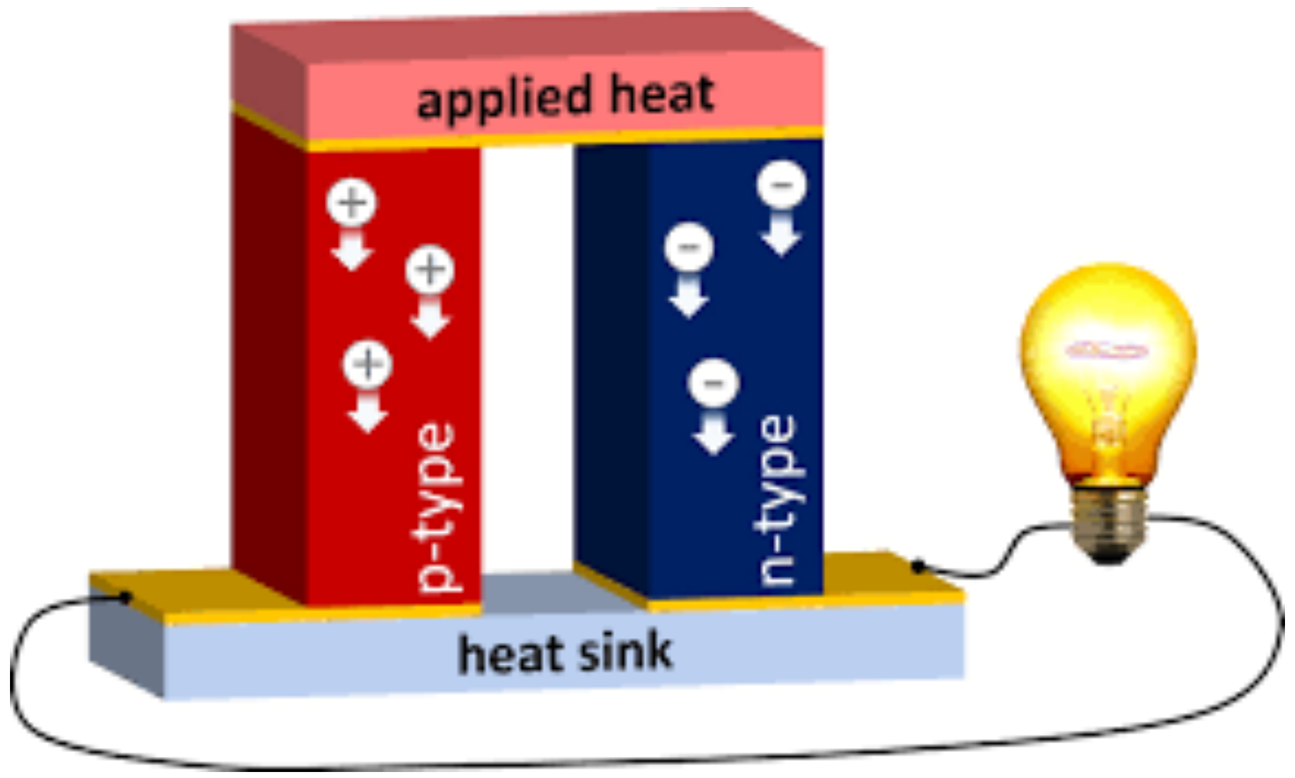
In this work, we study the transport of vorticity on curved dynamical two-dimensional magnetic membranes. We find that topological transport can be controlled by geometrically reducing symmetries, enabling processes absent from flat magnetic systems. To this end, the vorticity 3-current is constructed, which obeys a continuity equation immune to local disturbances of the magnetic texture and spatiotemporal fluctuations of the membrane. We show how electric current can manipulate vortex transport in geometrically nontrivial magnetic systems. As an illustrative example, we propose a minimal setup that realizes an experimentally feasible energy storage device and discuss its thermodynamic efficiency in terms of a *vortexoelectric* counterpart of the thermoelectric figure of merit ZT.



In basic textbook thermo-electrics:

- Thermo-electric functionality is based on cross-coupling of two conserved fluxes:

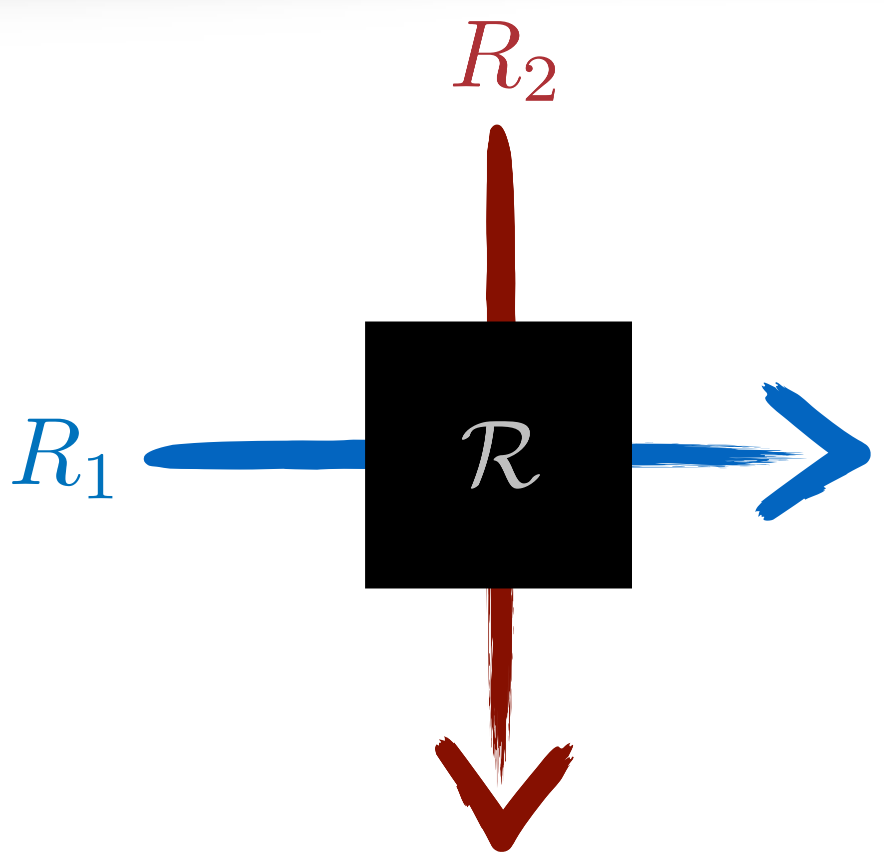




$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_1 & -\mathcal{R} \\ -\mathcal{R} & R_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

conjugate forces | Onsager-reciprocal response | fluxes

Underlying thermodynamic efficiency for various energy-transmutation devices



$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_1 & -\mathcal{R} \\ -\mathcal{R} & R_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

- Optimal thermodynamic efficiency for devices based on one quantity driving the other is governed by a dimensionless parameter

$$\xi = \frac{\mathcal{R}^2}{R_1 R_2}$$

"cooperativity" between the two fluxes

- Second law of thermodynamics requires the response matrix to be positive-semidefinite:

$$0 \leq \xi \leq 1$$

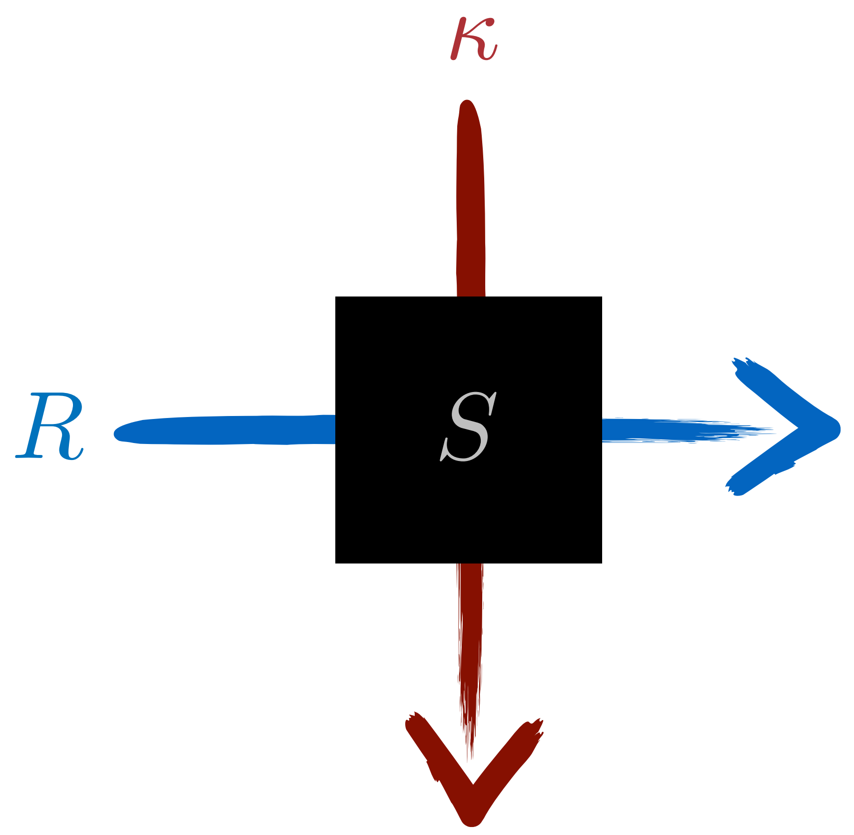
the upper limit (strong "cooperativity") corresponds to ideal Carnot machines

---

An illustrative example with a maximal efficiency is provided by a symmetric two-component fluid: $R_1 = R_2 = \mathcal{R}$

The associated dissipation in this case is given by the relative motion of the two fluids: $\sum V_i I_i = \mathcal{R}(I_1 - I_2)^2$

Thermo-electric efficiency is historically parametrized by dimensionless ZT



$$\begin{pmatrix} V \\ I_h \end{pmatrix} = \begin{pmatrix} R & -S \\ TS & \kappa \end{pmatrix} \begin{pmatrix} I_c \\ \Delta T \end{pmatrix}$$

- For historic reasons, thermoelectric efficiency is defined as

$$ZT \equiv \frac{S^2}{R\kappa} T \to \mathcal{Z}$$

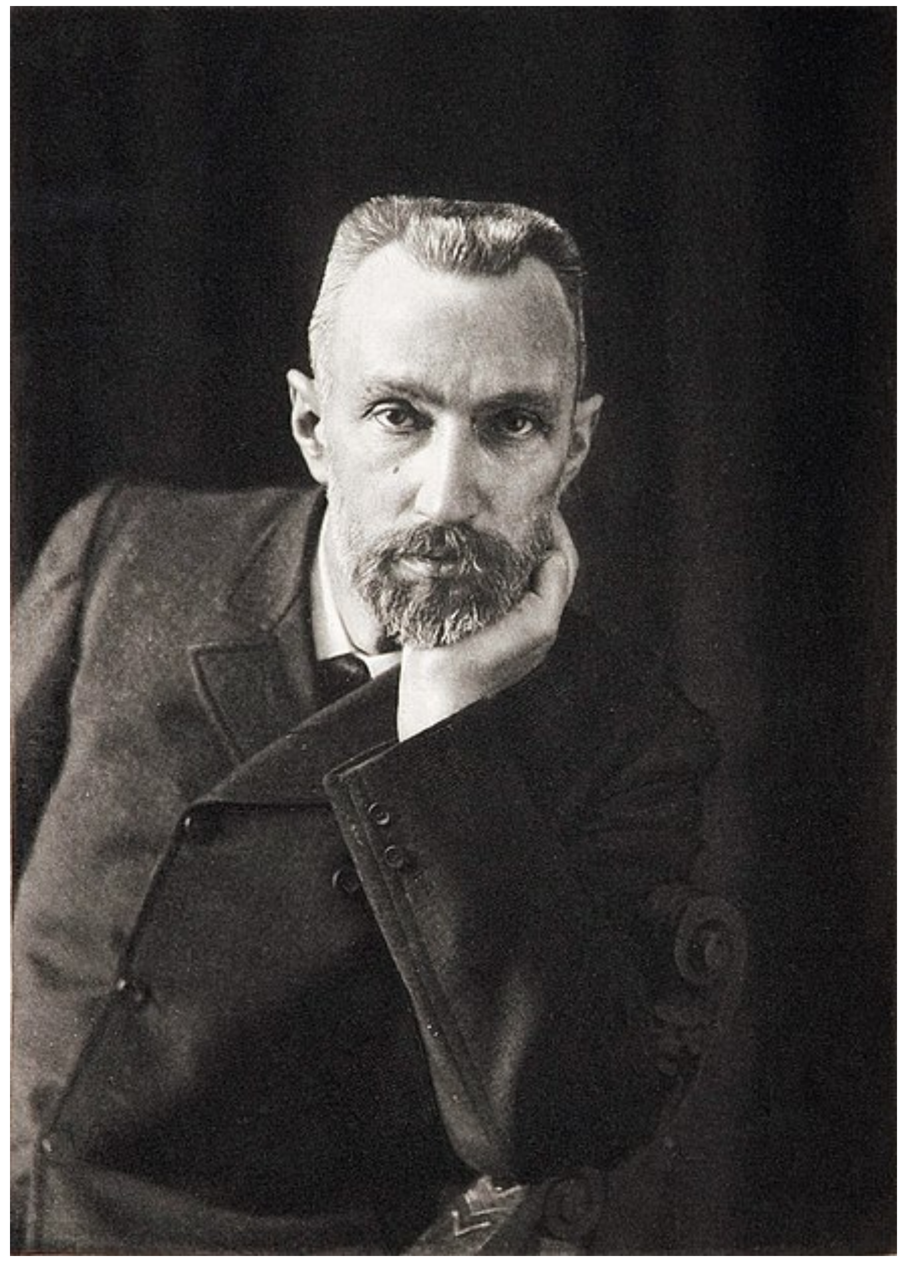
- This corresponds, in our notation, to

$$\mathcal{Z} \equiv \frac{\xi}{1-\xi}$$ which is bounded as $0 \le \mathcal{Z} \le \infty$ when $0 \le \xi \le 1$

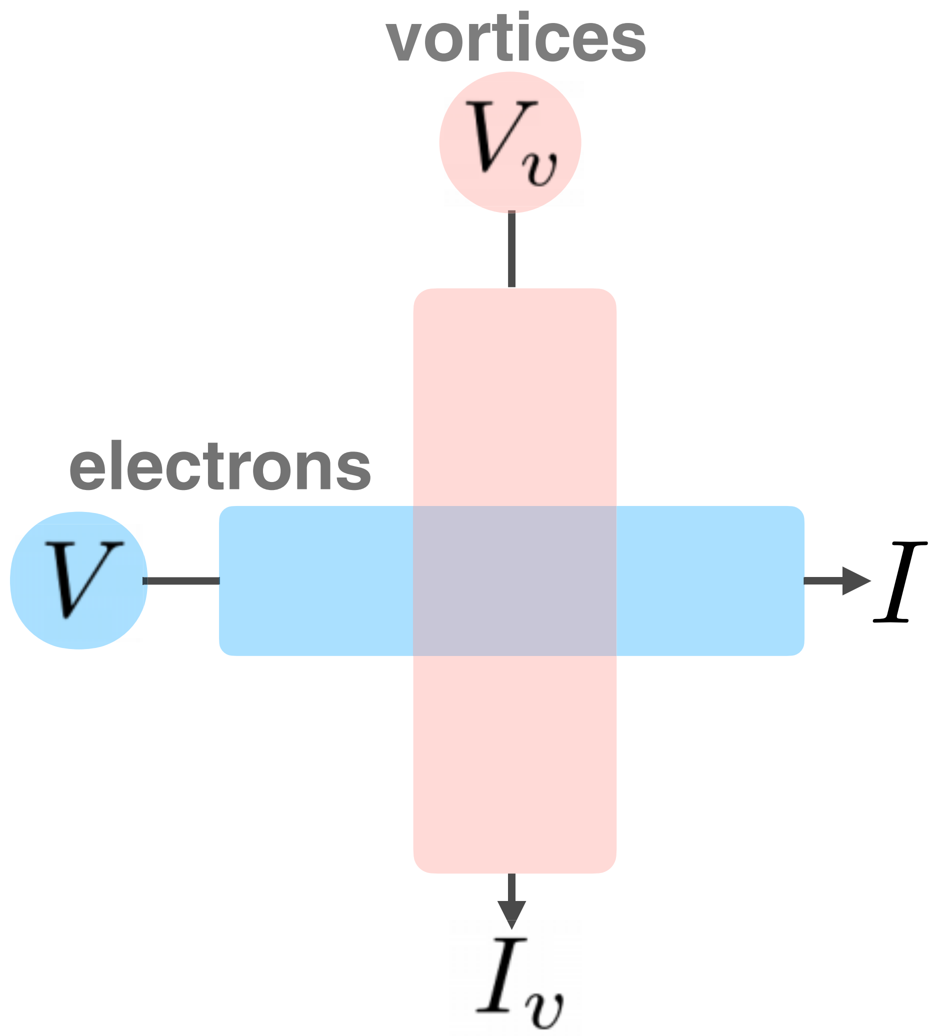
Examples: thermal power generator or electrical (Peltier) cooler - efficiencies of both are controlled by this same dimensionless *ZT* parameter

Such thermo-electrics lay out a general blueprint for “X-electrics”:

- We propose to explore other (topological) conservation laws in magnetic systems, giving rise to a similar cross-coupling with an electrical (or thermal) flux
- Topological energy storage as a natural (proof-of-principle) thermodynamic functionality
- Reducing symmetries to enable the cross-coupling using either magnetic order or geometry of the device



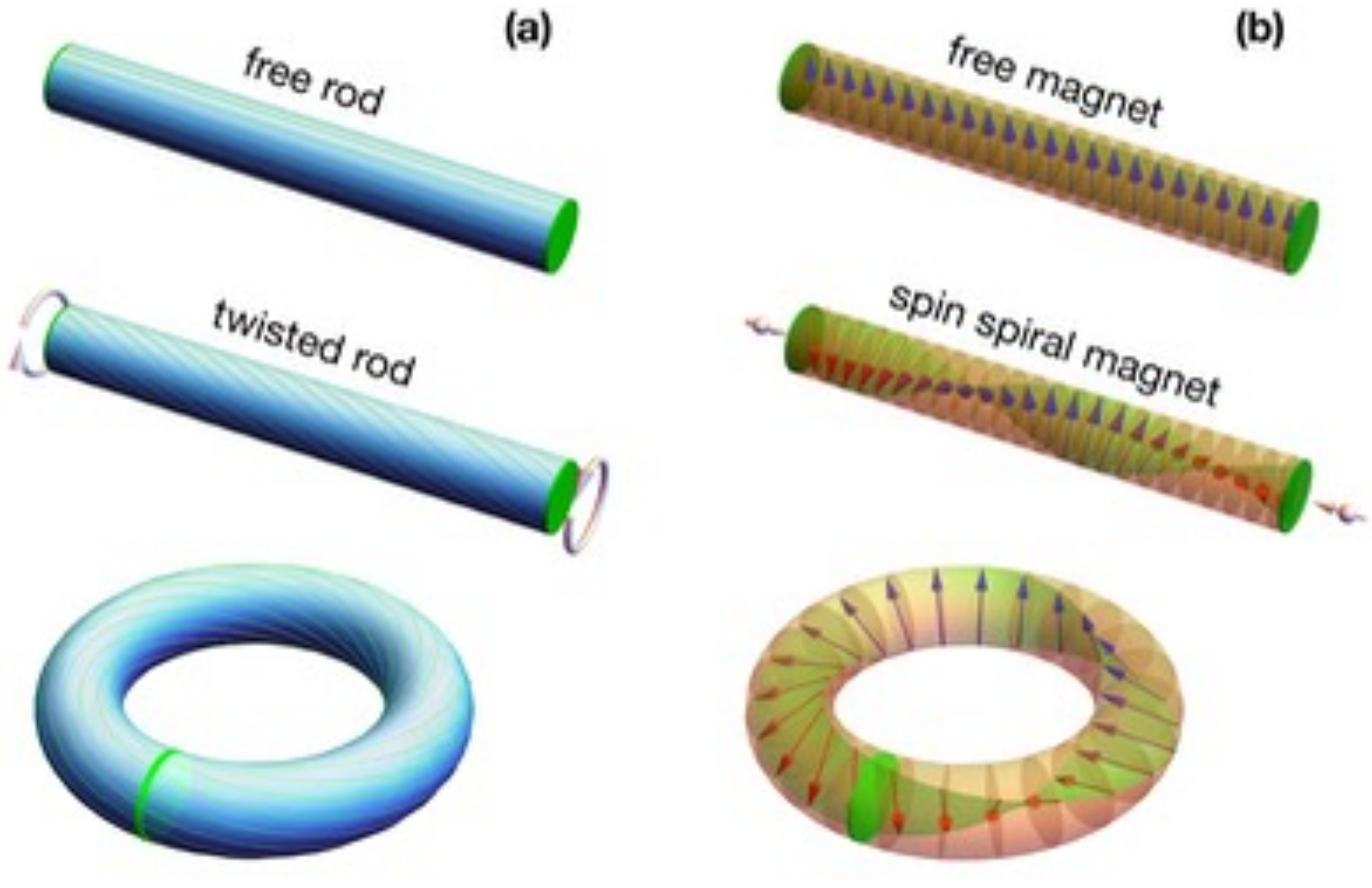
Pierre Curie:
“Asymmetry creates the phenomenon”

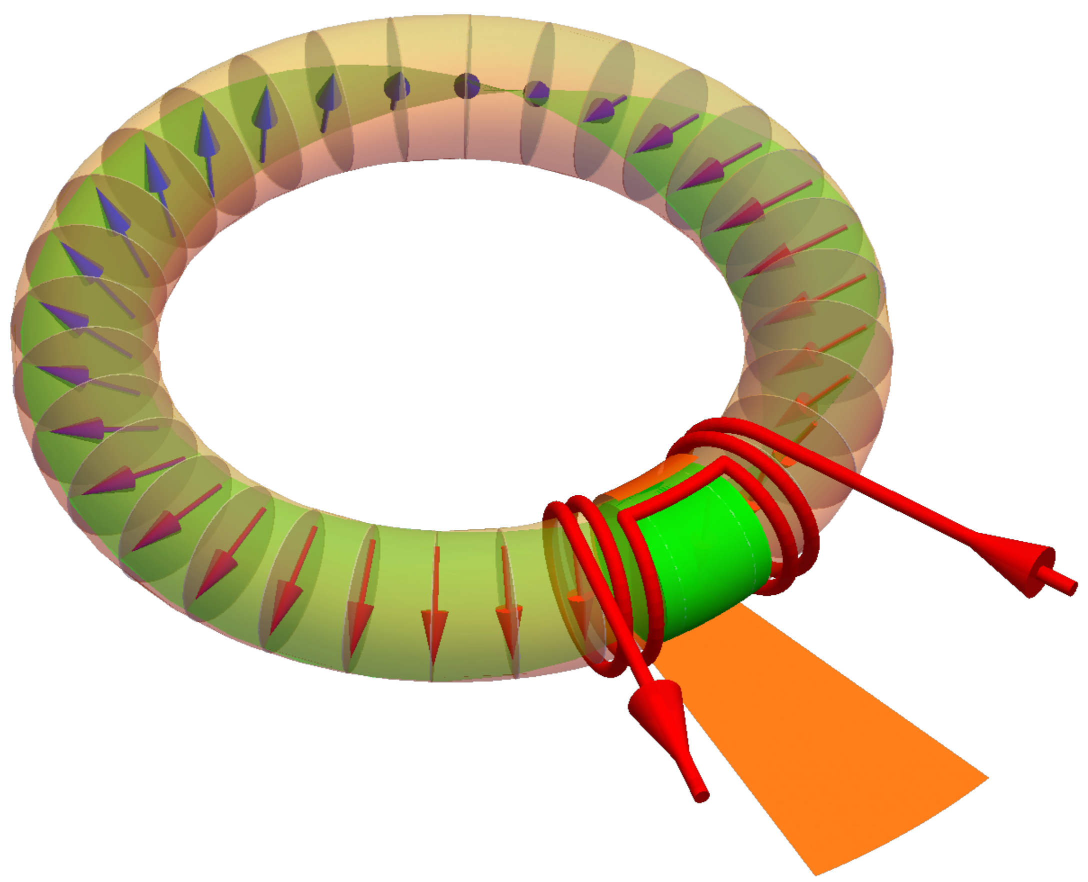


vortexo-electrics

- as a potential practical application, we propose to utilize the resultant vortex motion for building up magnetic winding for energy storage
- thermodynamic efficiency is independent of the actual application

Our original topological energy-storage concept left some open questions

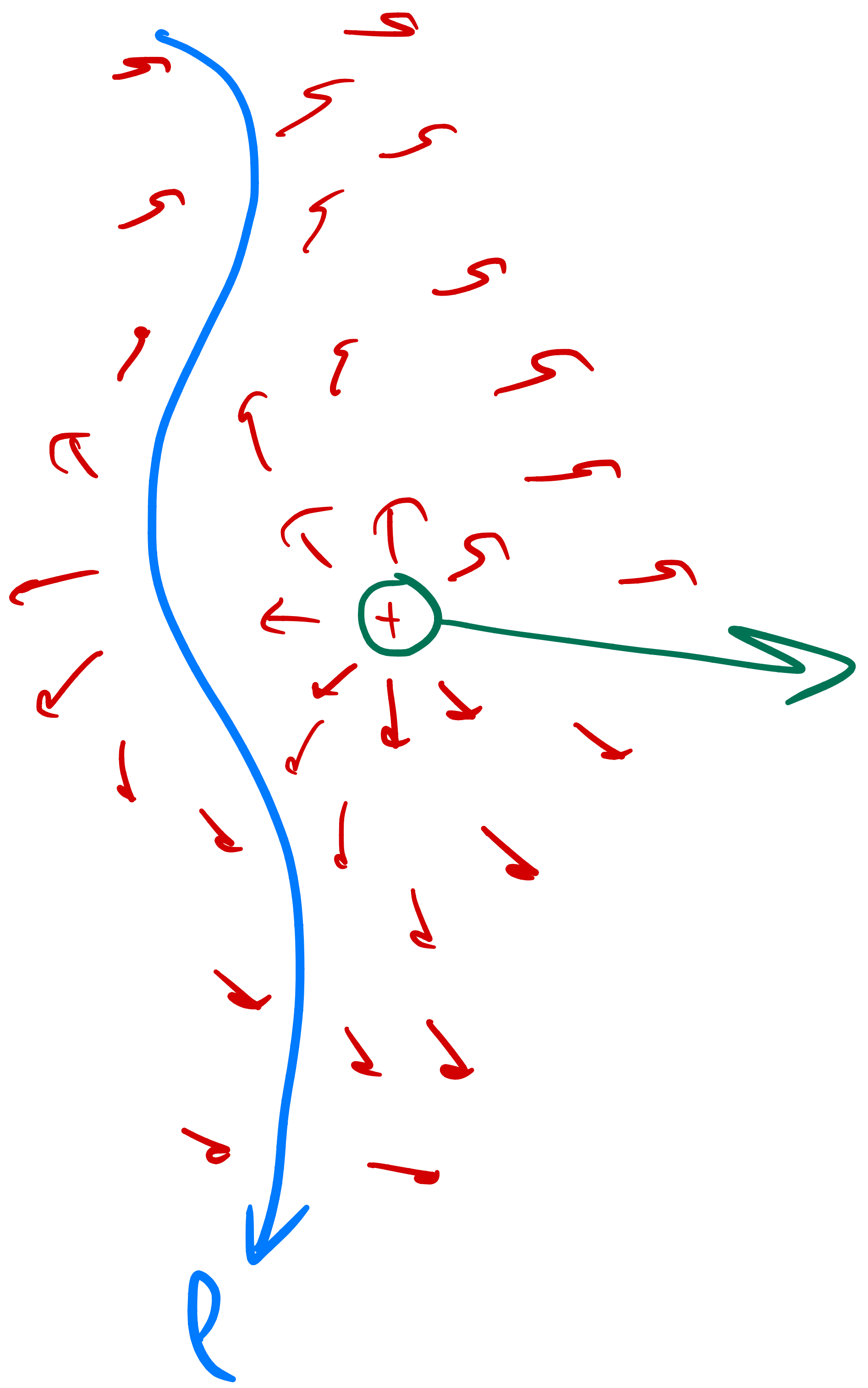




- How to inject/extract winding in practice?
- Boundary (twisting) vs bulk (vorticity)
- Controlling symmetries: Heterostructure design vs curved geometry
- Thermodynamic efficiency?

YT and Xiao, *PRL* (2018)

The more appealing approach is based on extensive force on vortices:



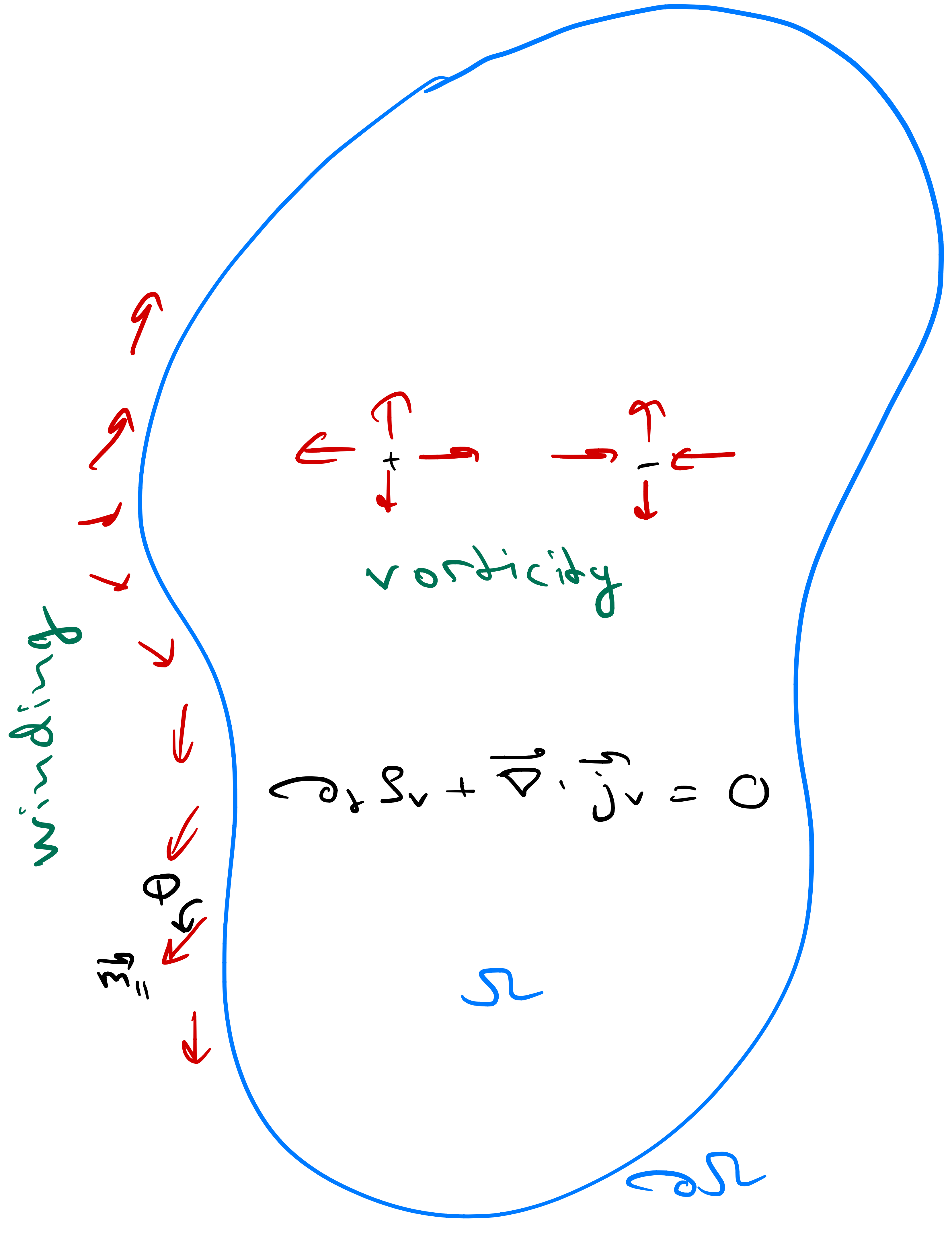
vorticity flow transverse to ℓ

⇕

winding change along ℓ

- If we establish practical means to push vorticity within a 2D system, winding would build up in the transverse direction (cf. phase slips in 2D superfluids)
- This links up our energy-storage agenda with the **transport of vorticity**: recasting winding buildup as a transport phenomenon

Recall vorticity-winding transmutation along any edge/boundary:



Net winding:

$$Q = \oint_{\partial\Omega} \frac{d\Phi}{2\pi} \quad \vec{m}_{\parallel}^{\,z}$$

Stokes $\rightarrow$ $\int_{\Omega} d^2r \;\; \rho_v$

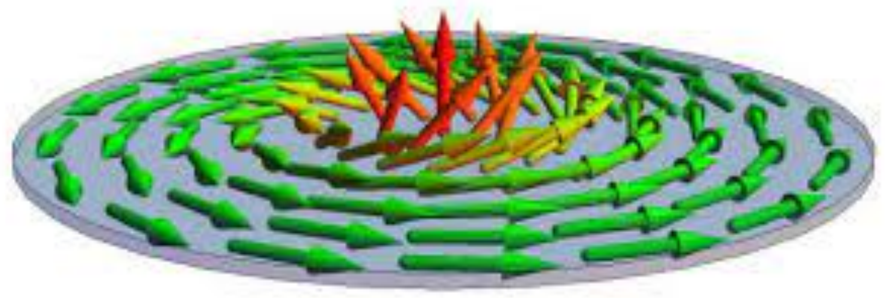
where $\rho_v = \dfrac{\hat{z}\cdot\partial_x \vec{m}\times\partial_y\vec{m}}{4\pi}$

vorticity density

$$j_v^{(i)} = \frac{\epsilon^{ij} \;\; \hat{z}\cdot\partial_i \vec{m}\times\partial_t\vec{m}}{4\pi}$$

vorticity flux

topological charge Q labels winding homotopy classes in the strongly-ordered *XY* limit

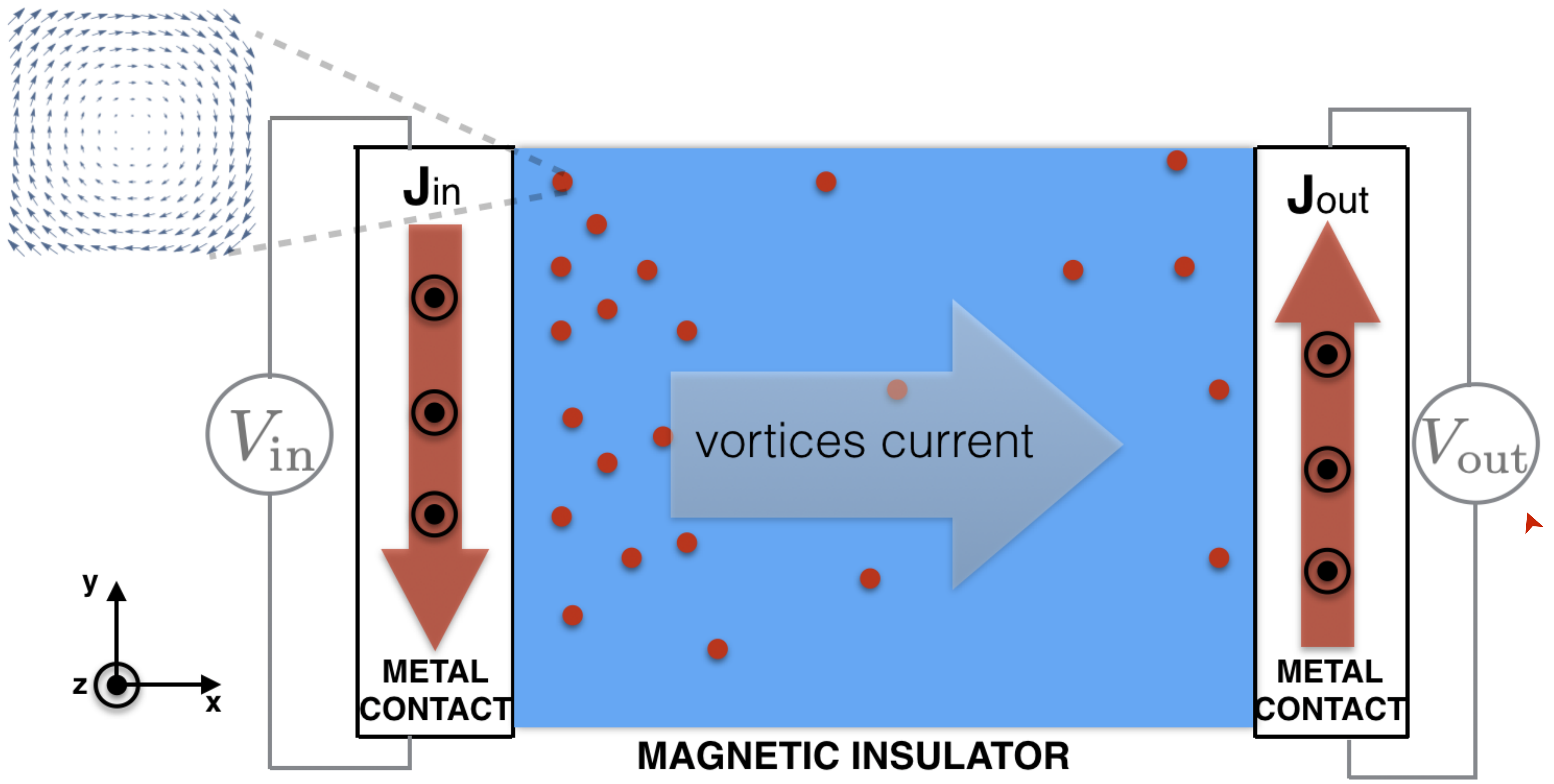


$$Q = \int d^2r\, \rho_v \rightarrow 1$$

net integrated vorticity is naturally *conserved* and tends to be *charge-neutral* over any extended region

Recall the "electrical Magnus force" on vortices:

Energetics (thermodynamics) and symmetries:



spin torque by the input current at the left interface:

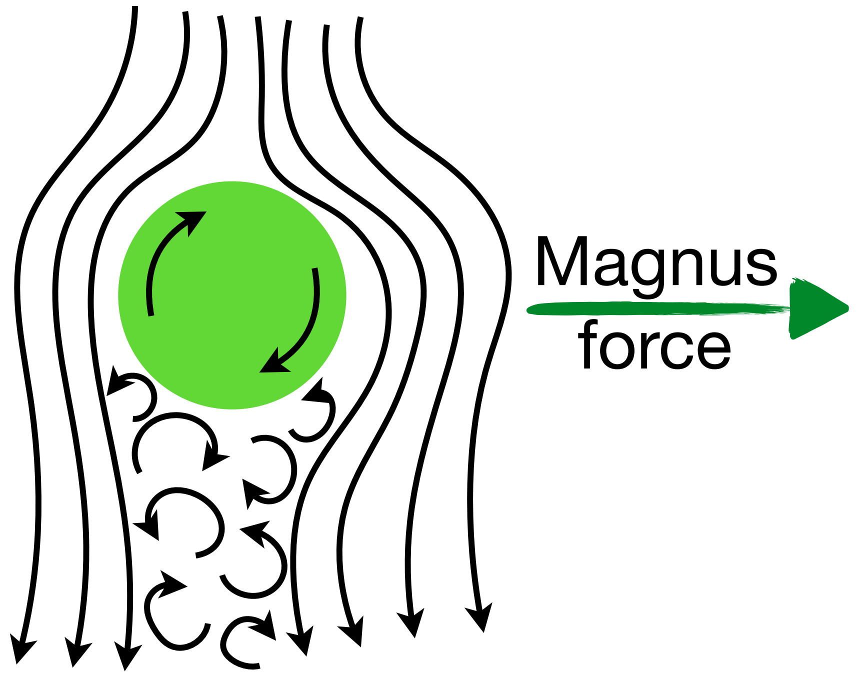
$$\boldsymbol{\tau} = \eta \mathbf{M} \cdot \mathbf{m}(\mathbf{J} \cdot \boldsymbol{\nabla})\mathbf{m}$$

$\mathbf{M} \propto \mathbf{z}$ - magnetization of the metal contact

$\mathbf{m}(x, y, t)$ - magnetic texture of the insulator

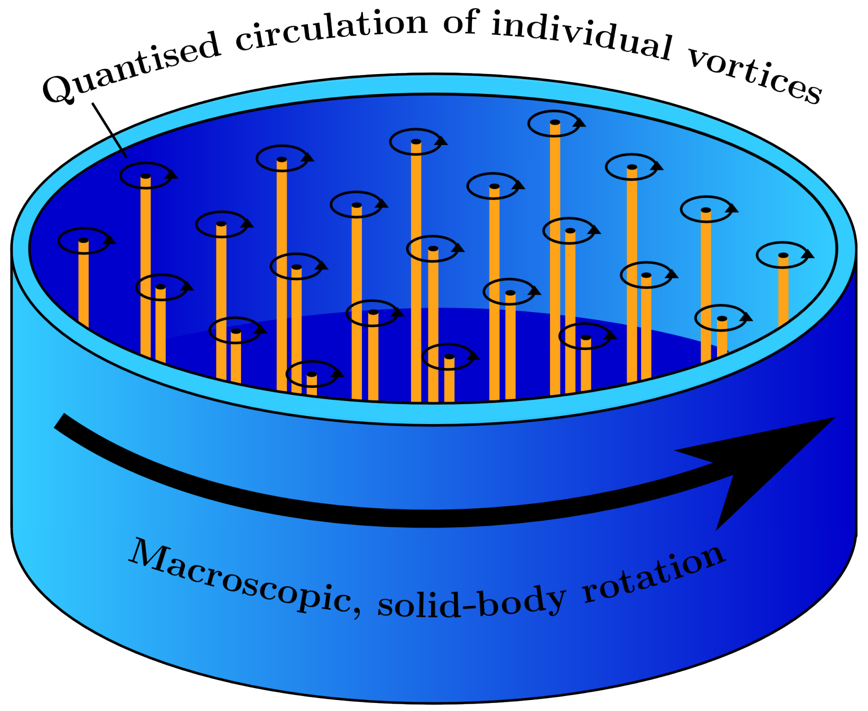
Onsager-reciprocal motive force

Zou, Kim, and YT, *PRB* (2019)

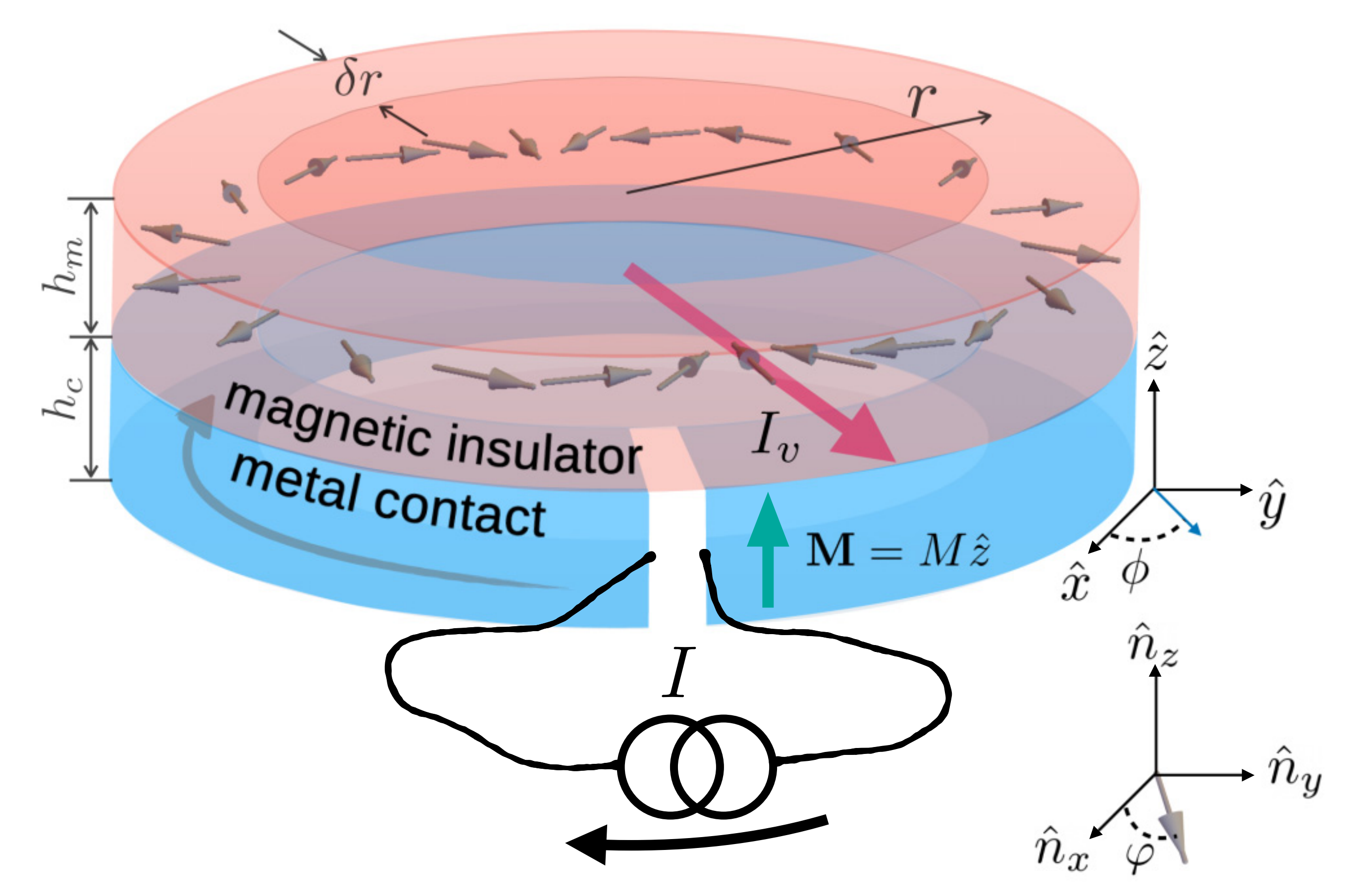


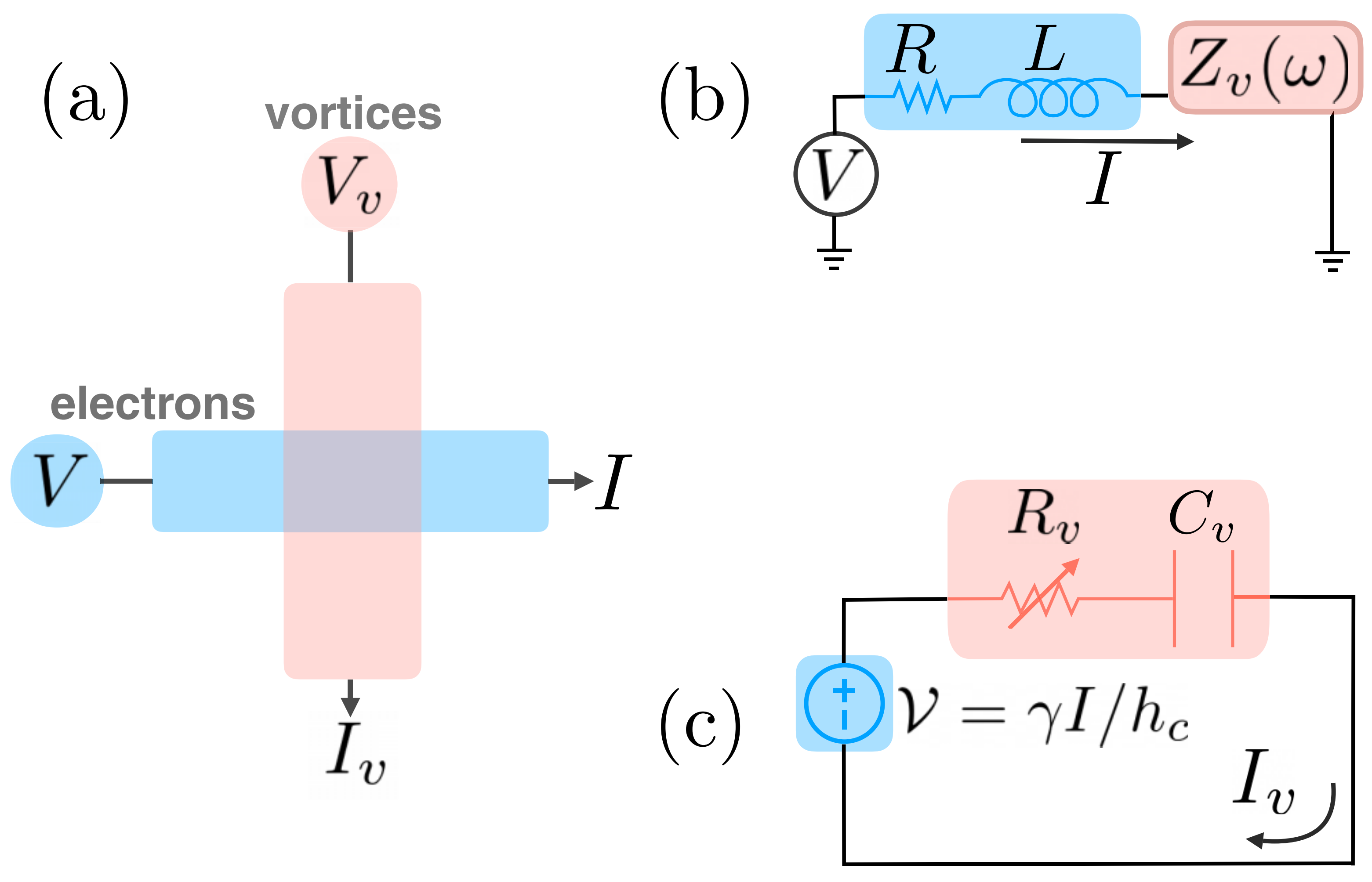
$$\dot{W} = \int dy\, \boldsymbol{\tau} \cdot (\mathbf{m} \times \partial_t \mathbf{m}) \rightarrow \eta\, \mathbf{z} \cdot \mathbf{J} \times \mathbf{j}$$

charge current vortex flux



Corbino geometry for vortex transport turns this into energy storage:





- Vortex and electron fluxes cross-couple via Magnus-like friction:

$$\begin{pmatrix} V \\ V_v \end{pmatrix} = \begin{pmatrix} R & \mathfrak{R} \\ -\mathfrak{R} & R_v \end{pmatrix} \begin{pmatrix} I \\ I_v \end{pmatrix}$$

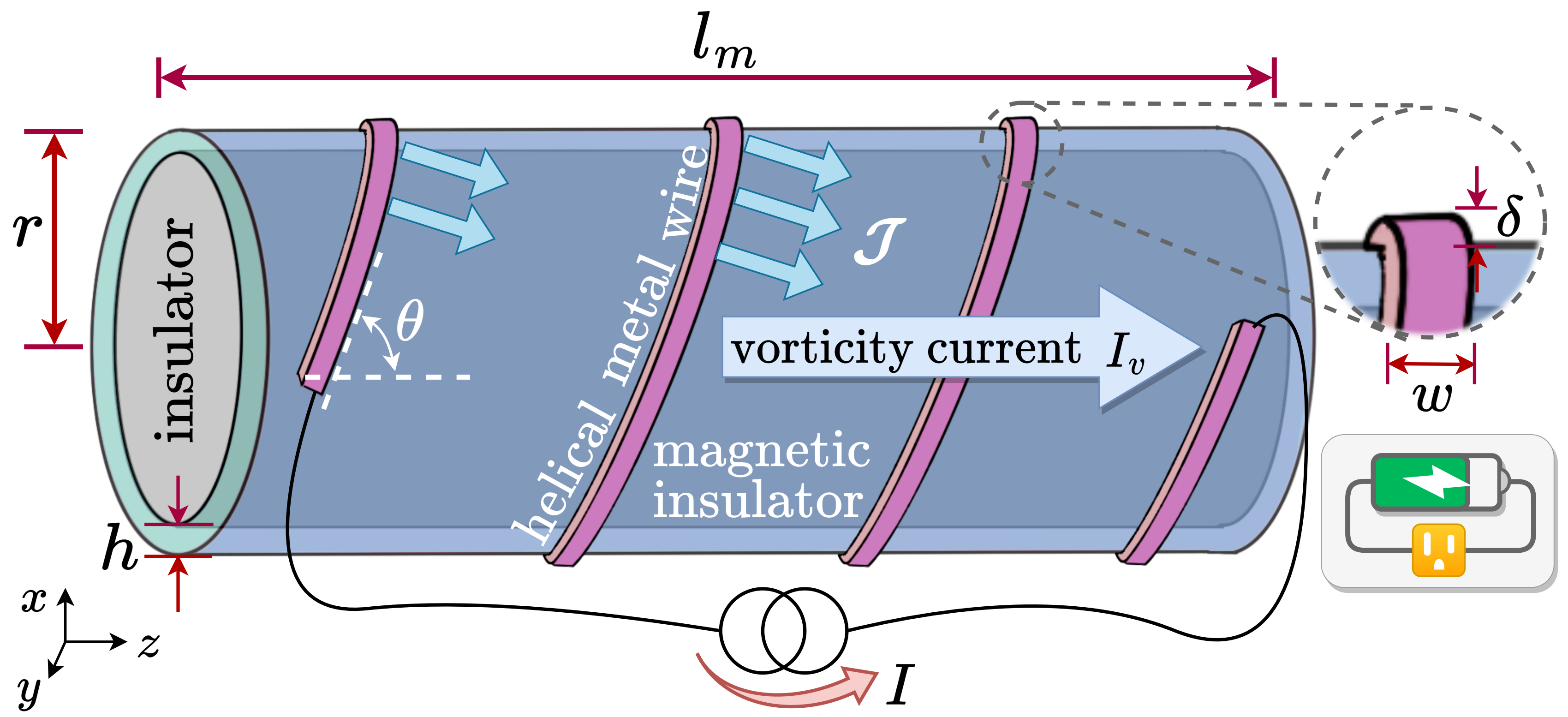
- The magnetic annulus acts as a winding capacitor:

$$I_v = \frac{dQ_w}{dt} \qquad E_v = \frac{Q_w^2}{2C_v} \qquad C_v \propto \text{exchange}^{-1}$$

- It is tempting to think of devices where a tunable vortex conductivity (e.g., associated with their binding and/or defect pinning) is utilized as a switch

Jones, Zou, Zhang, and YT, *PRB* (2020)

Alternatively: Geometry-enabled topological energy storage



- electric current along the spiral drives vorticity flow along the cylinder axis, thus building up uniform transverse winding
- note an analogy with CISS (chirality-induced spin selectivity)

spin-transfer input power: $$\dot{W} = \int dl\, \boldsymbol{\tau} \cdot \mathbf{m} \times \partial_t \mathbf{m} \propto \mathfrak{T}\, I I_v$$

$\mathfrak{T}$ - (pseudoscalar) “torsion of a curve” (electrical wire), which can effectively (from the symmetry point of view) convert the local geometric normal to the surface into an out-of-plane magnetization

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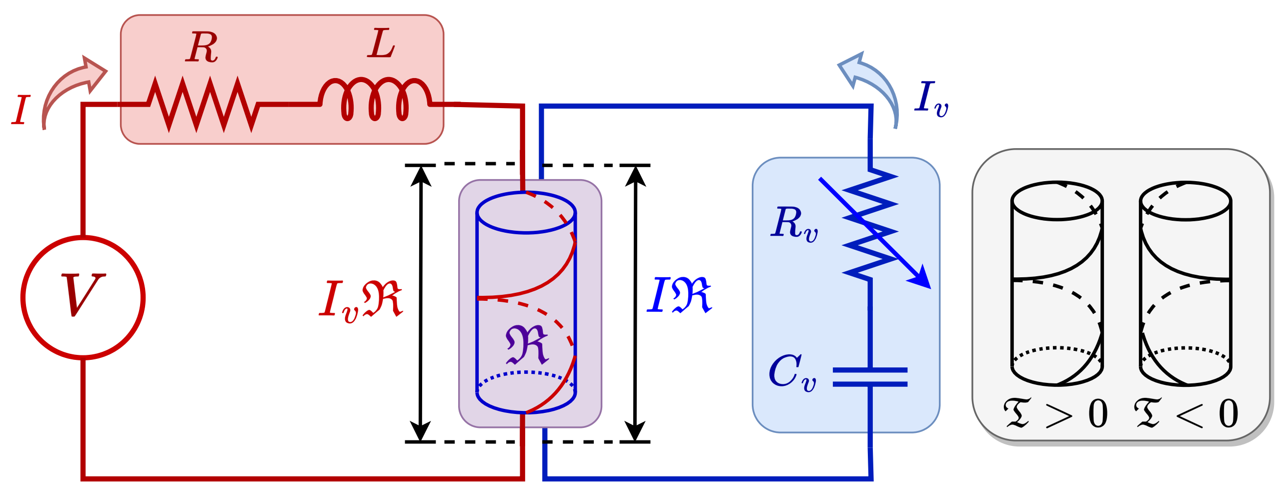


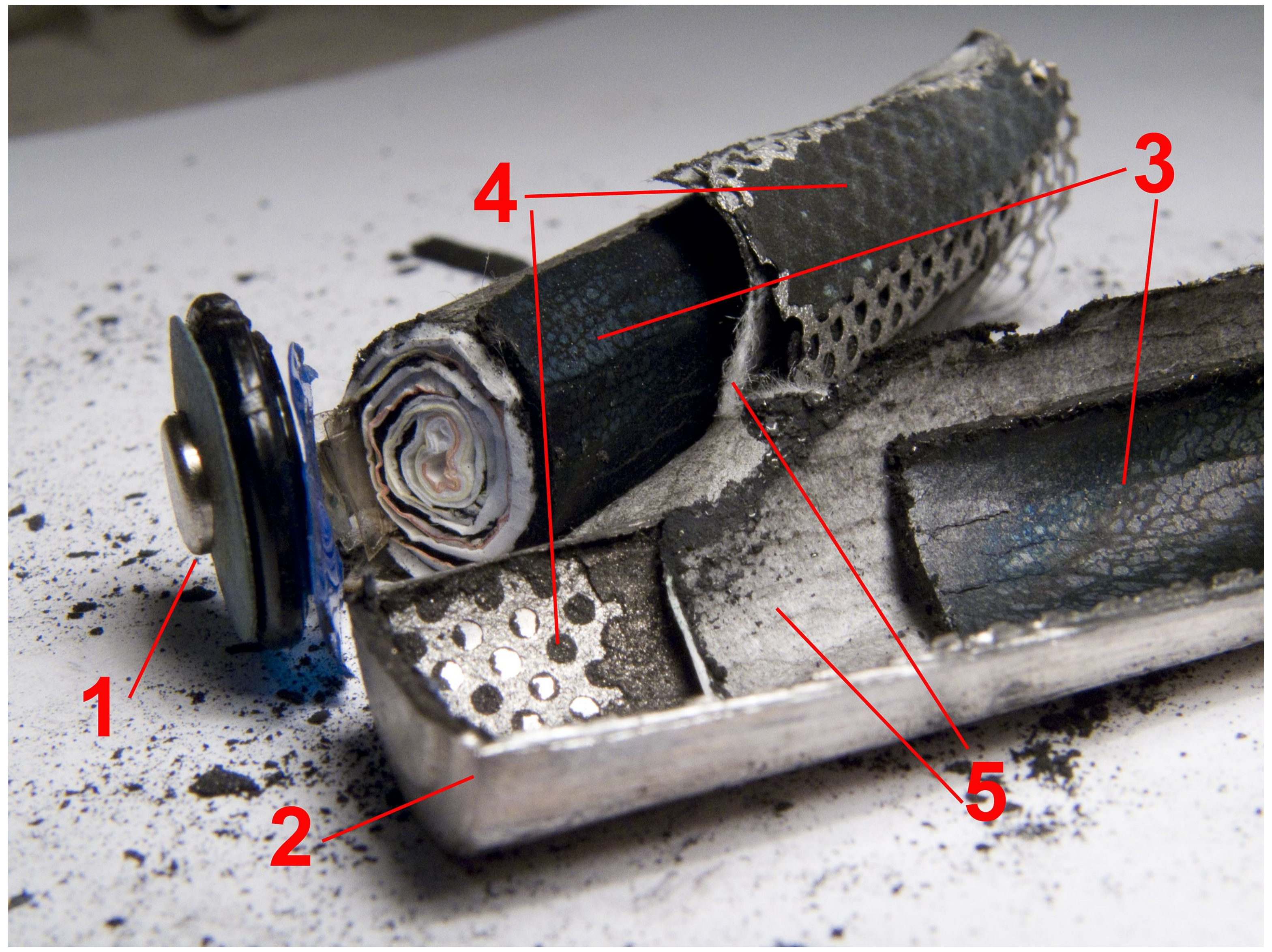
$$\xi \propto \mathfrak{T}^2 / R R_v$$

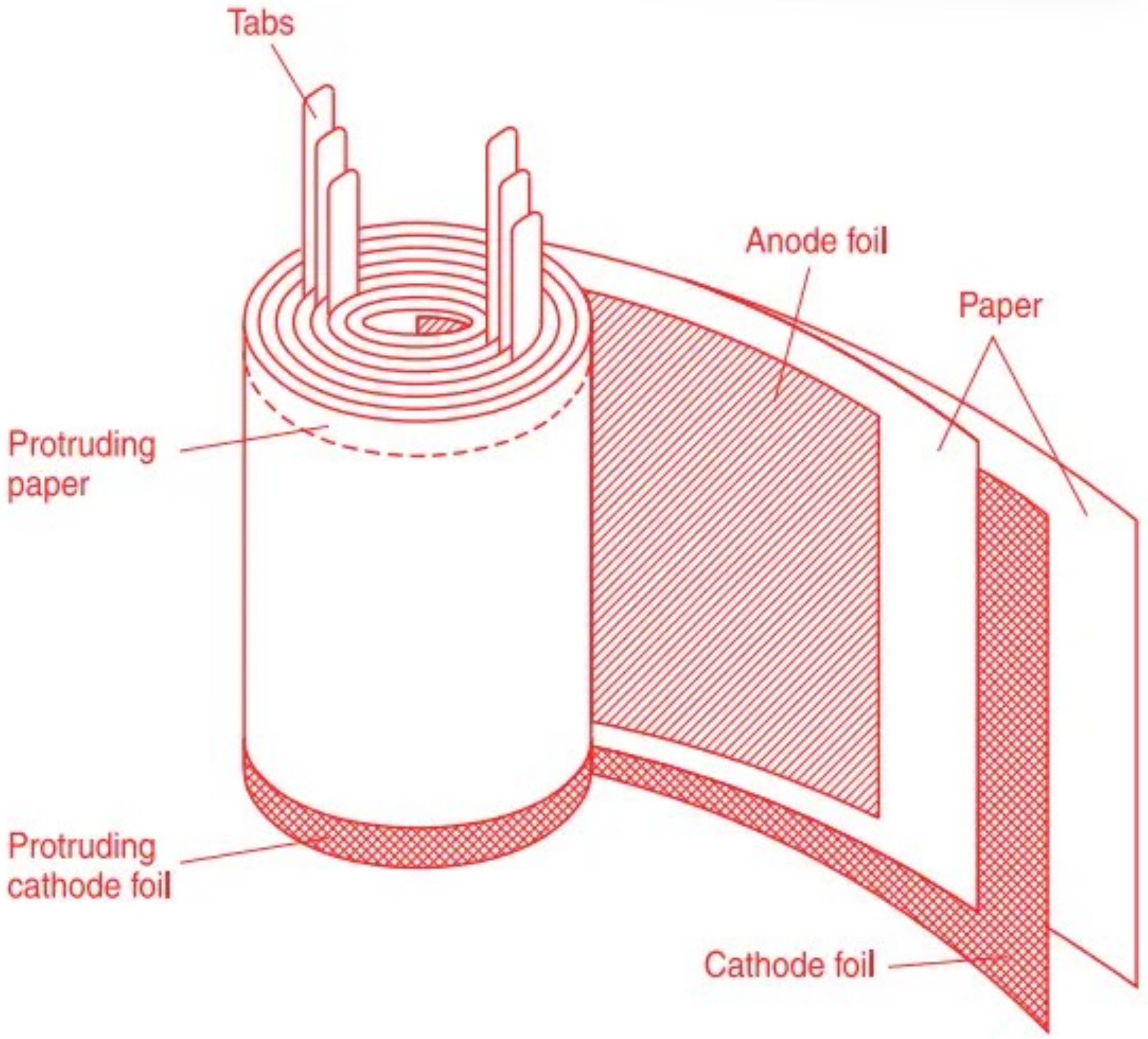
electron-vortex “cooperativity”

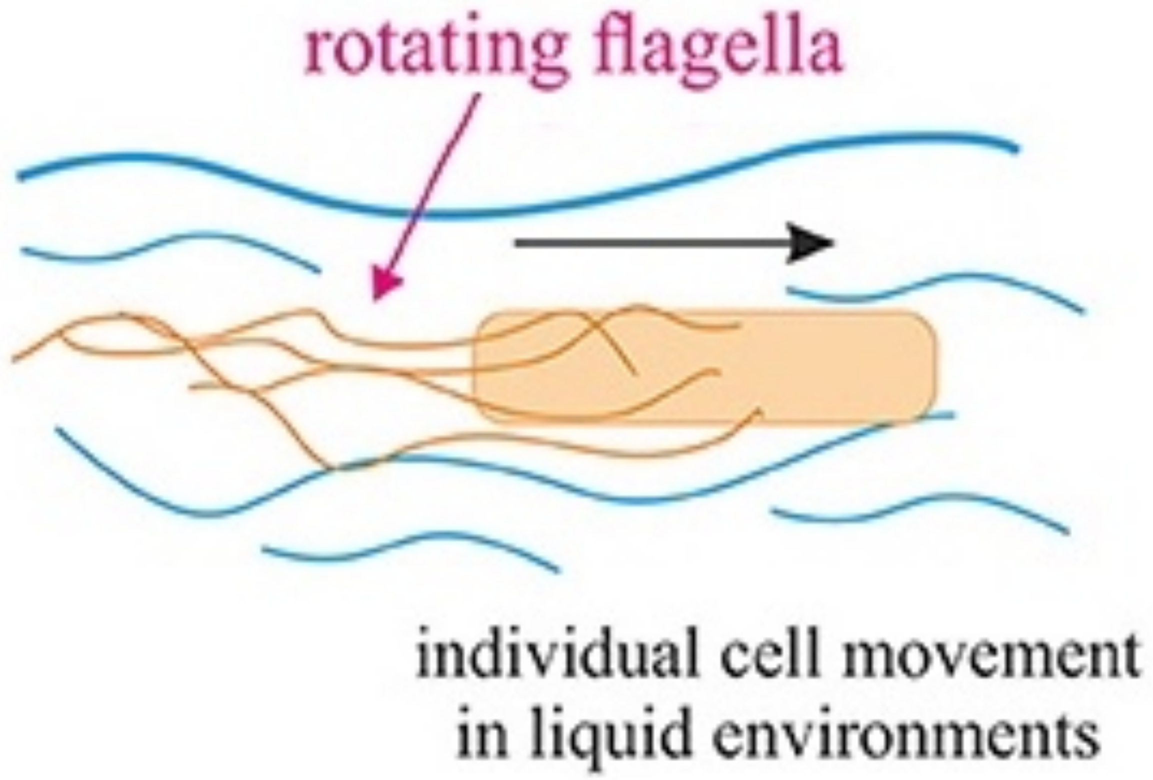
the effective dimensionless parameter, which is thermodynamically bounded to [0,1], is formally analogous to the thermoelectric figure of merit *ZT*

Dao, Zou, Kleinherbers, and YT, *PRB* (2025)

Some inspiration from existing storage technologies and the nature

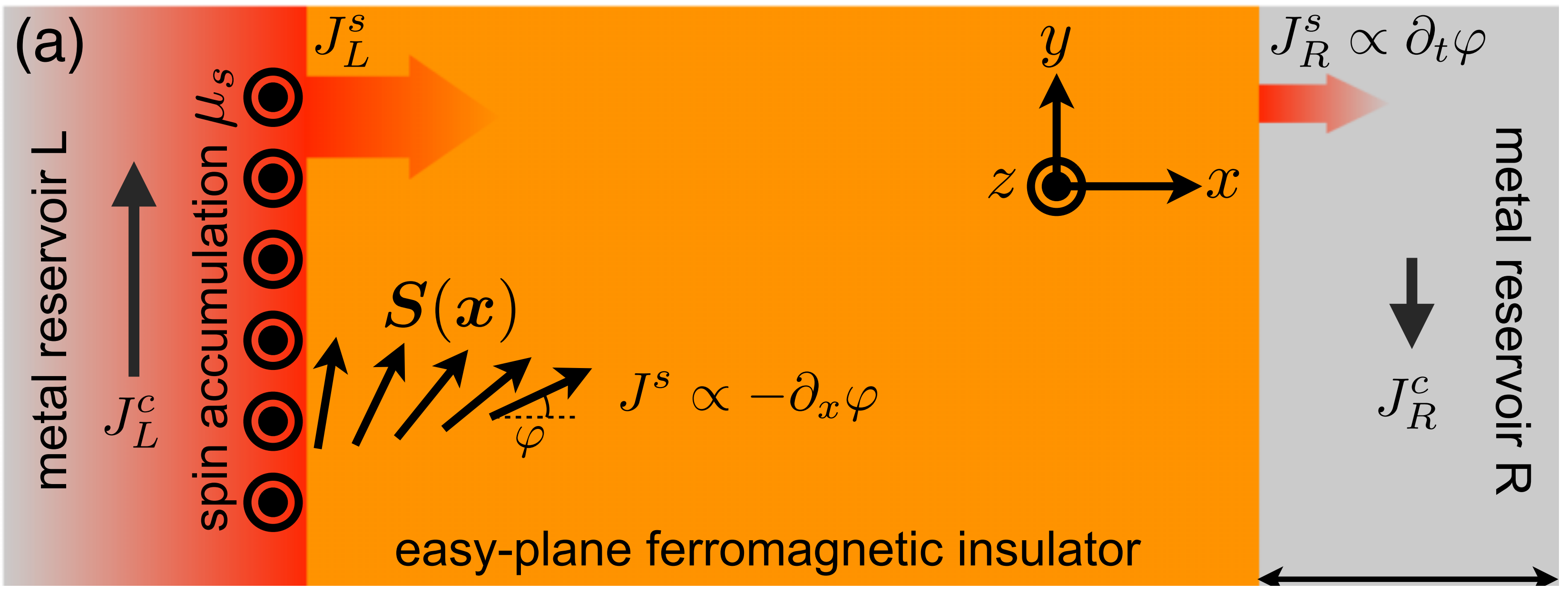






- Bacterial motility: ability to swim using metabolic energy
- Each single flagellum (helical appendage) has a rotary motor at the base that can turn clock- or anticlockwise
- Small Reynolds number: Viscosity-dominated hydrodynamics; thus, continuous transduction of chemical into mechanical energy

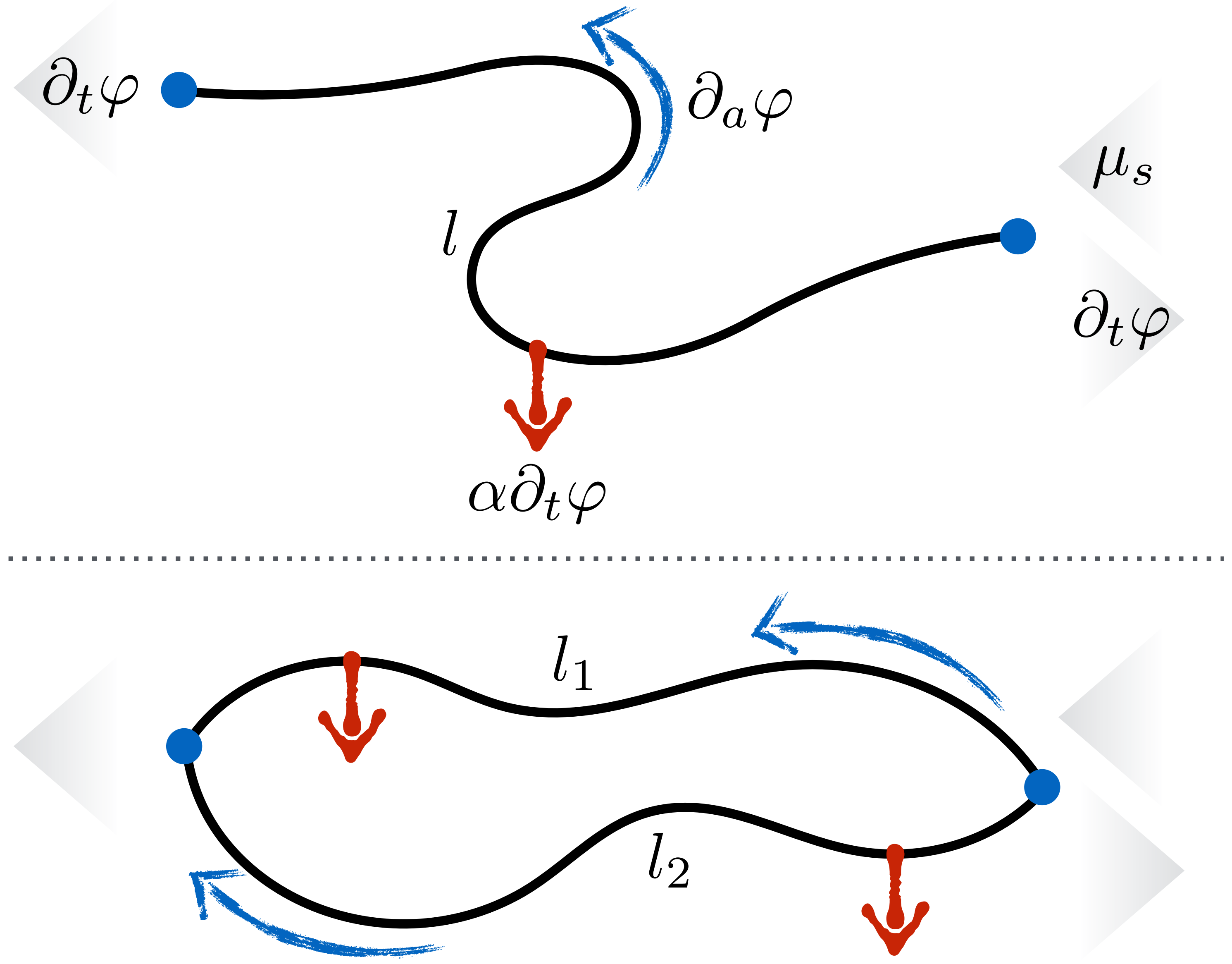
Switching gears: Let us utilize macroscopic coherence and nonlinearity



$$\{\varphi(x), \rho_s^{(z)}(x')\} = \delta(x - x')$$

Poisson bracket in full analogy with superconductivity

Takei and YT, *PRL* (2014)

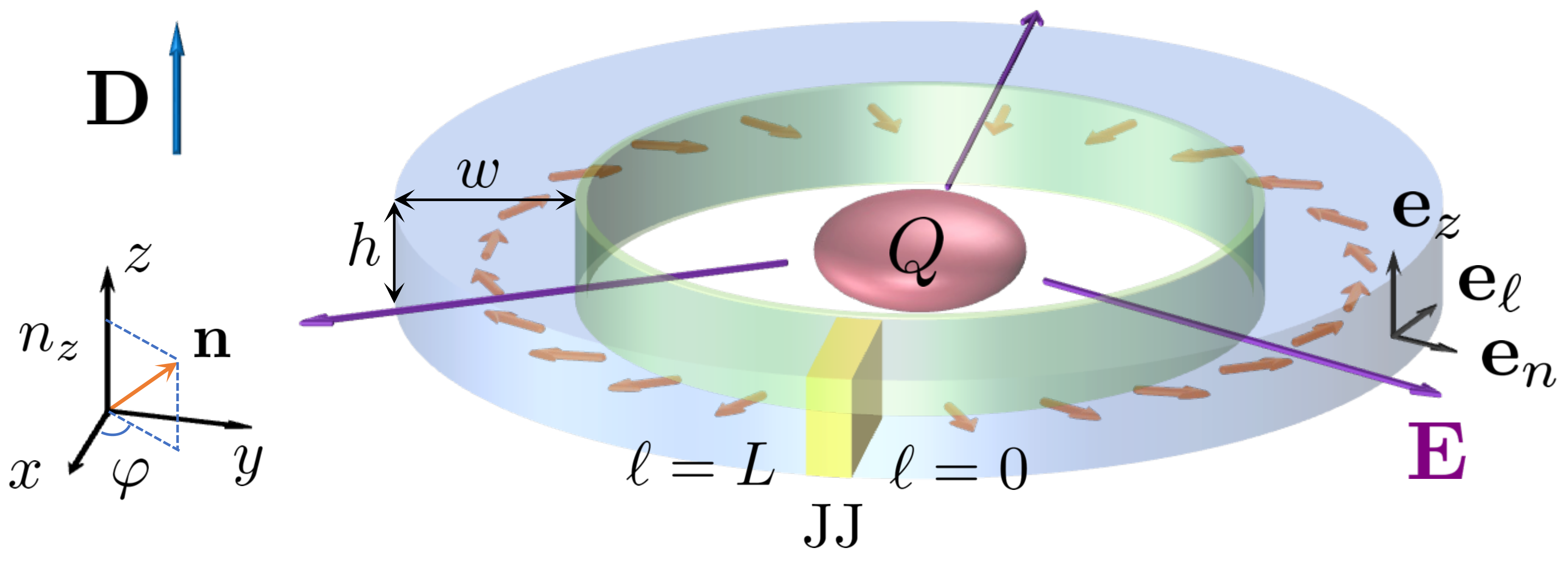


YT and Kläui, *PRL* (2017)

- Linear (weak-texture) 1D spin-superfluid dynamics have a purely *hydrodynamic character* with no aspects of coherence

- In the nonlinear regime, the (Landau-type) superflow instability is sensitive to *interference in the loop geometry*, on the lengthscale set by the coherence (i.e., healing) length

Nonlinearities are enhanced by introducing weak links

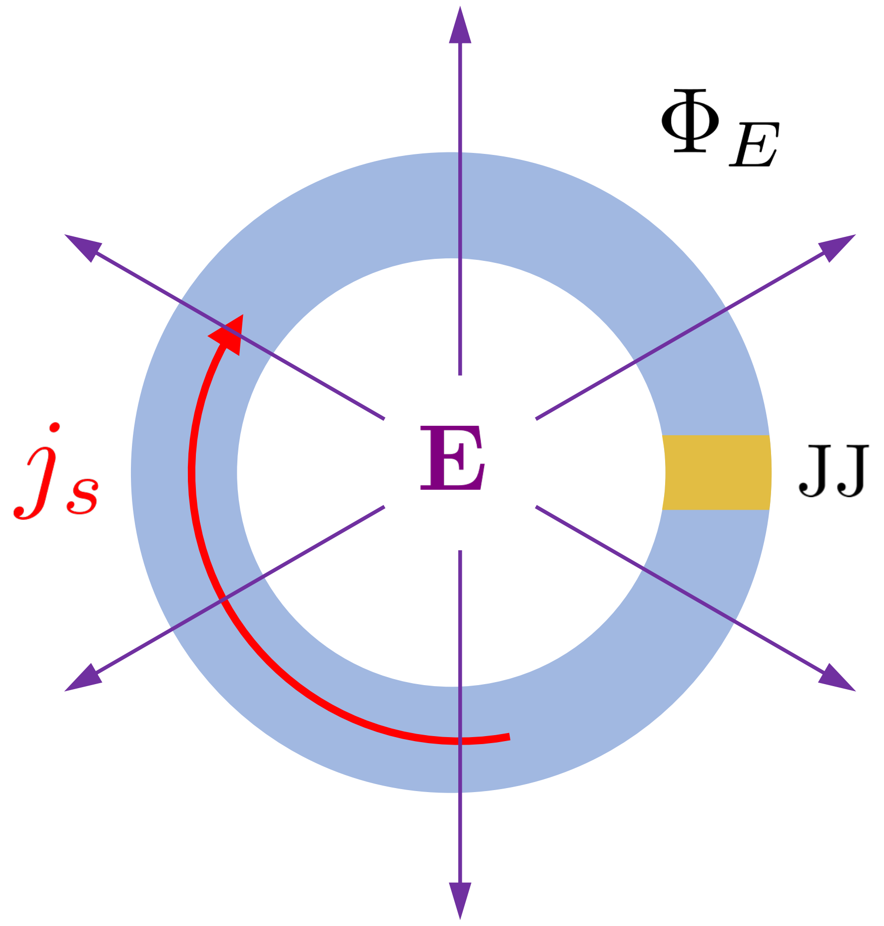


key nonlinearities are associated with the Josephson junction dynamics

- *Phase slips* in the weak link are a hallmark of Josephson junction physics and functionalities
- The dynamics in the magnetic ring, away from the weak link should be well-described by the well-controlled low-energy (spin-superfluid) *hydrodynamics*
- The (macroscopic) coherent structure of the superflow is manifested via the spin current flowing through the weak link, which is sensitive to the net radial *electric field flux* through the loop, opening prospects for *electric-field sensing*

Zhu, Kleinherbers, Levitov, and YT, *PRB* (2025)

A proposal for spin rf SQUID for electric-field sensing



rf SQUID with a single
Josephson junction

- The gauge field, rooted microscopically in the electronic spin-orbit interaction, is micromagnetically manifested as a *Dzyaloshinski-Moriya interaction*
- Electric-field detection is proposed as a dual to the magnetic field detection in a superconducting rf SQUID
- The rf SQUID modality of the device is focusing on its *(nonlinear) microwave characteristics*, without a need for depositing electrical contacts for spin injection or detection

Zhu, Kleinherbers, Levitov, and YT, *PRB* (2025)

Superfluid helium *interferometers*

Yuki Sato and Richard Packard

Emerging devices for measuring quantum phase offer a possible new window into phenomena far outside condensed-matter physics.

Figure 1. Two fluid reservoirs coupled through a small pipe and subjected to a pressure difference *P*. **(a)** In the classical case, fluid flows in one direction from the higher-pressure side to the lower-pressure one. **(b)** But in the quantum case, superfluid oscillates at the pipe at a frequency proportional to *P*. **(c)** In the apparatus used to observe the effect, an array of apertures (red) serves as a weak link separating the two superfluid reservoirs. A flexible diaphragm serves as a pressure pump to try to force liquid through the weak link. The fluid oscillation is picked up by a sensing coil, or microphone. It can be readily heard through inexpensive headphones or measured more precisely by analyzing its power spectrum.

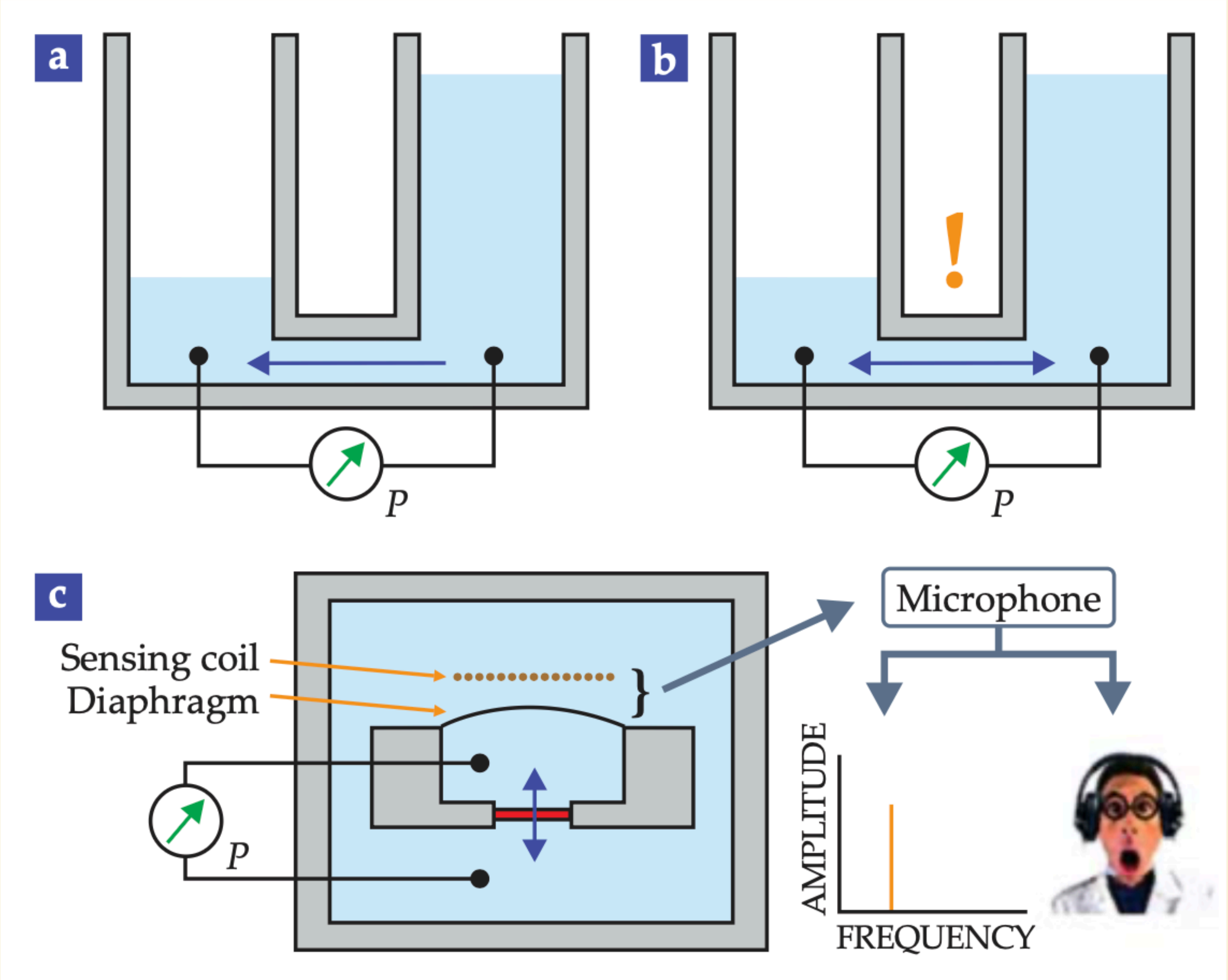


Figure 2. A superfluid interferometer. Two nanoaperture arrays (red) interrupt a torus filled with superfluid helium. In response to a pressure difference, fluid in both arrays oscillates at the same frequency. A gradient in the quantum phase within the torus alters the relative phase of the oscillations, which affects the total oscillation amplitude detected by the sensing coil.

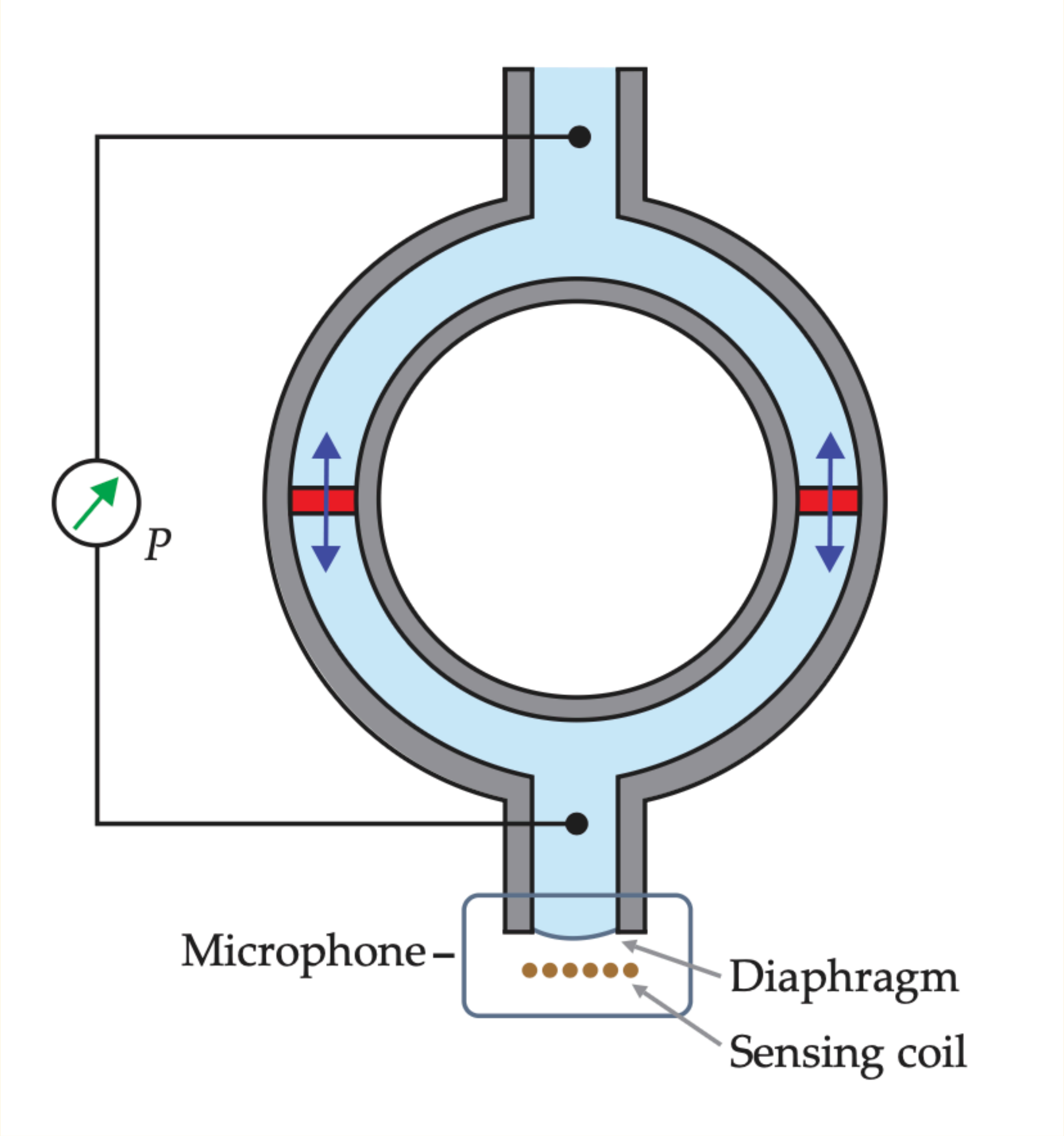
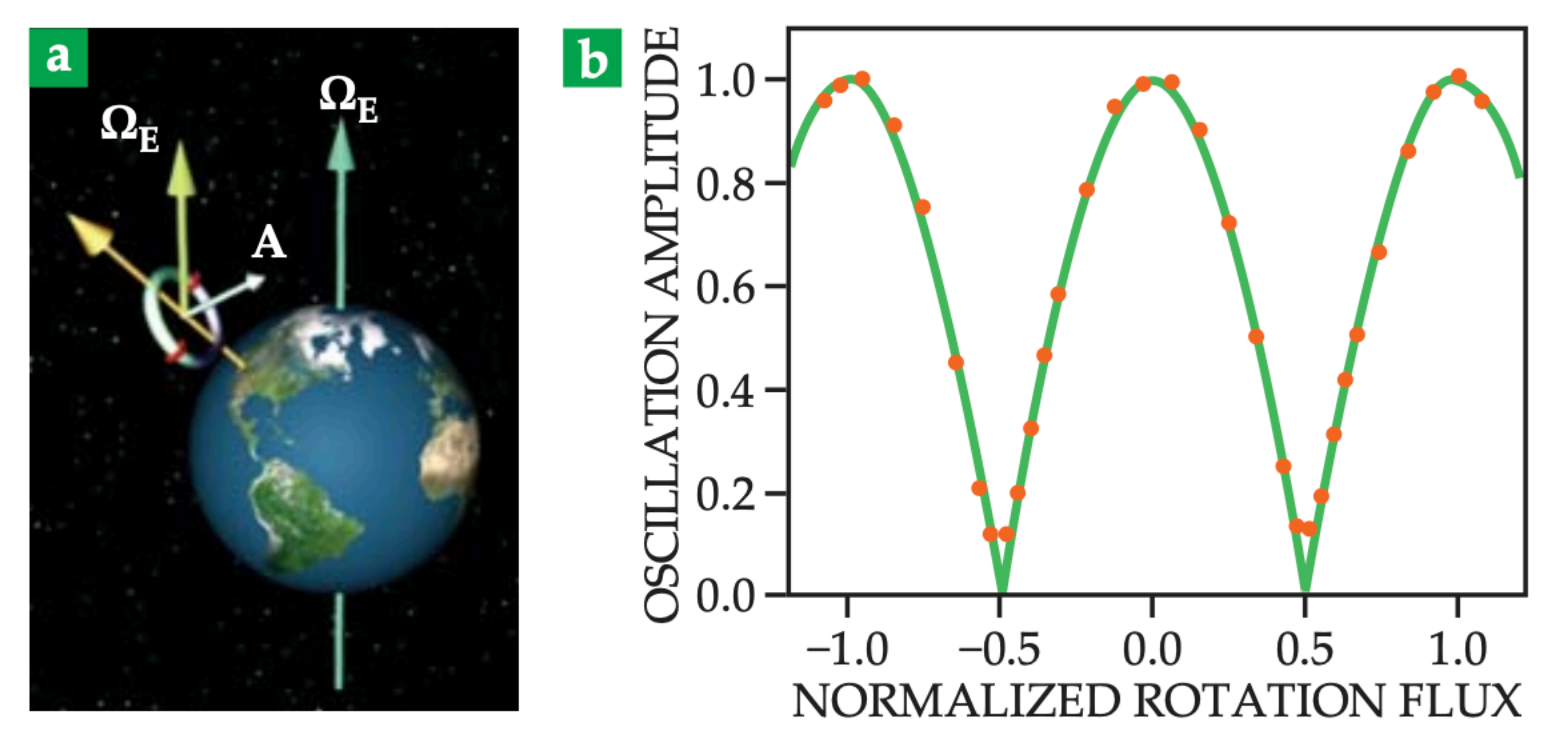
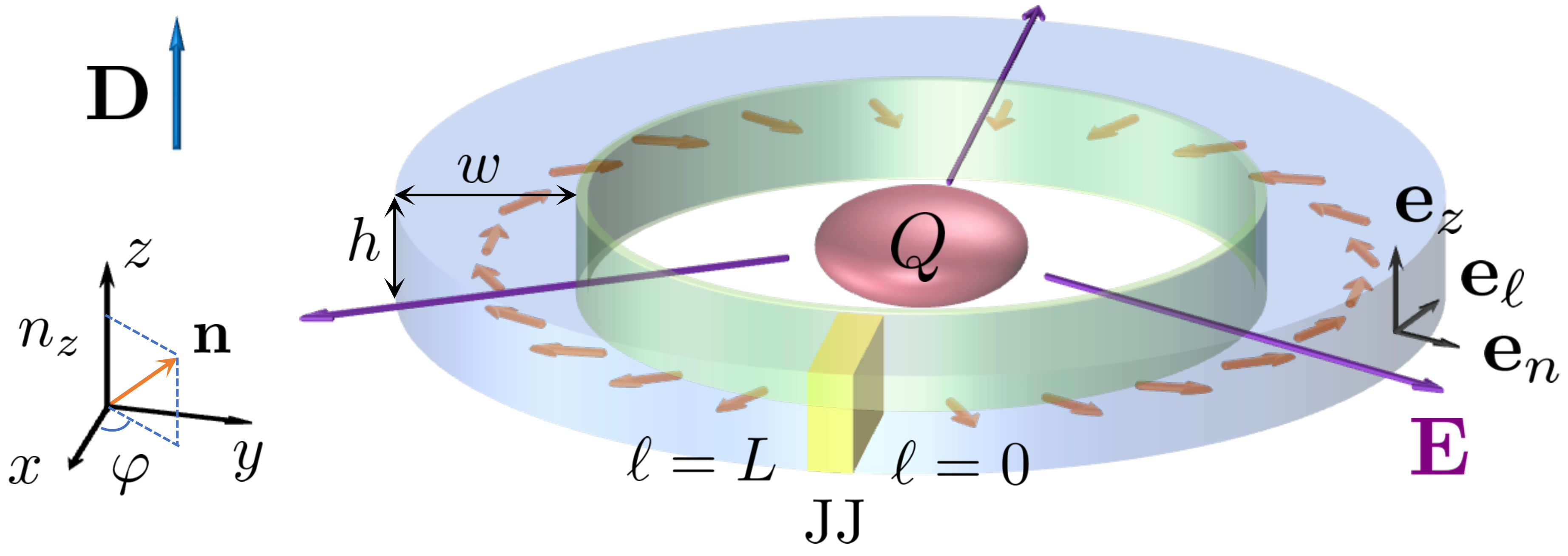


Figure 3. A superfluid gyroscope. (a) A superfluid interferometer is placed in Earth's rotating frame with its area vector **A** at some angle with respect to Earth's rotation vector $\boldsymbol{\Omega}_E$. The quantum phase shift across the device is proportional to the so-called rotation flux, $\boldsymbol{\Omega}_E \cdot \mathbf{A}$, so by varying the direction of **A** one can measure $\boldsymbol{\Omega}_E$. **(b)** The modulation in the oscillation amplitude, experimentally observed here with a helium-3 interferometer, has the form of the absolute value of a cosine. (Adapted from ref. 9.)



Basic energetics and dynamics of an rf spin SQUID



free energy functional to be fed into the LLG formalism:

$$\mathcal{F} = \int_0^L d\ell \left[\frac{A}{2}(\partial_\ell \mathbf{n})^2 + \frac{K}{2}n_z^2 - \mathbf{D}\cdot(\mathbf{n}\times\partial_\ell \mathbf{n})\right] - F_J \mathbf{n}(0)\cdot\mathbf{n}(L)$$

DMI: $\mathbf{D} \propto \mathbf{E}\times\mathbf{e}_l$

superfluid bulk dynamics:

$$\mathcal{F} = \int_0^L d\ell \left[\frac{A}{2}(\partial_\ell\varphi - a_E)^2 + \frac{K}{2}n_z^2\right] - J\cos(\varphi_L - \varphi_0)$$

$$\{\varphi(\ell), sn_z(\ell')\} = \delta(\ell - \ell')$$

canonical Poisson bracket

AC gauge field: $a_E \propto \mathbf{E}\cdot\mathbf{e}_n$

JJ boundary condition (spin-current continuity): $0 = \Delta\varphi - \phi_E + \beta\sin\Delta\varphi$

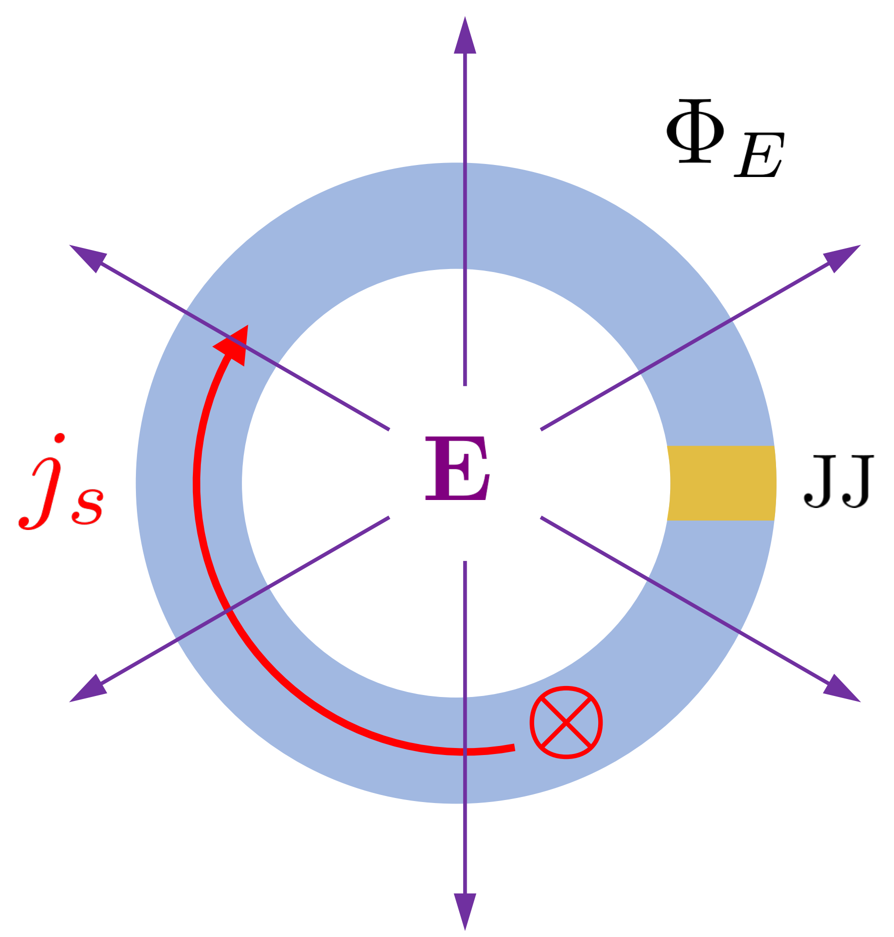
electric flux: $\phi_E = \int dl\, a_E$

$$\boxed{\beta \equiv \frac{JL}{A}}$$

the key dimensionless parameter that characterizes nonlinear properties and functionalities of the rf SQUID

Zhu, Kleinherbers, Levitov, and YT, *PRB* (2025)

Microscopics: The (Aharonov-Casher) gauge field



- *Microscopically*, a (linear in momentum) Pauli/Rashba interaction couples electric field to spin current

non-Abelian gauge field

$$H_{\text{Rashba}} = \frac{p^2}{2m} + \lambda \epsilon^{ijk} p_i \boxed{E_j \hat{\sigma}_k}$$

spin current
coupled to electric field

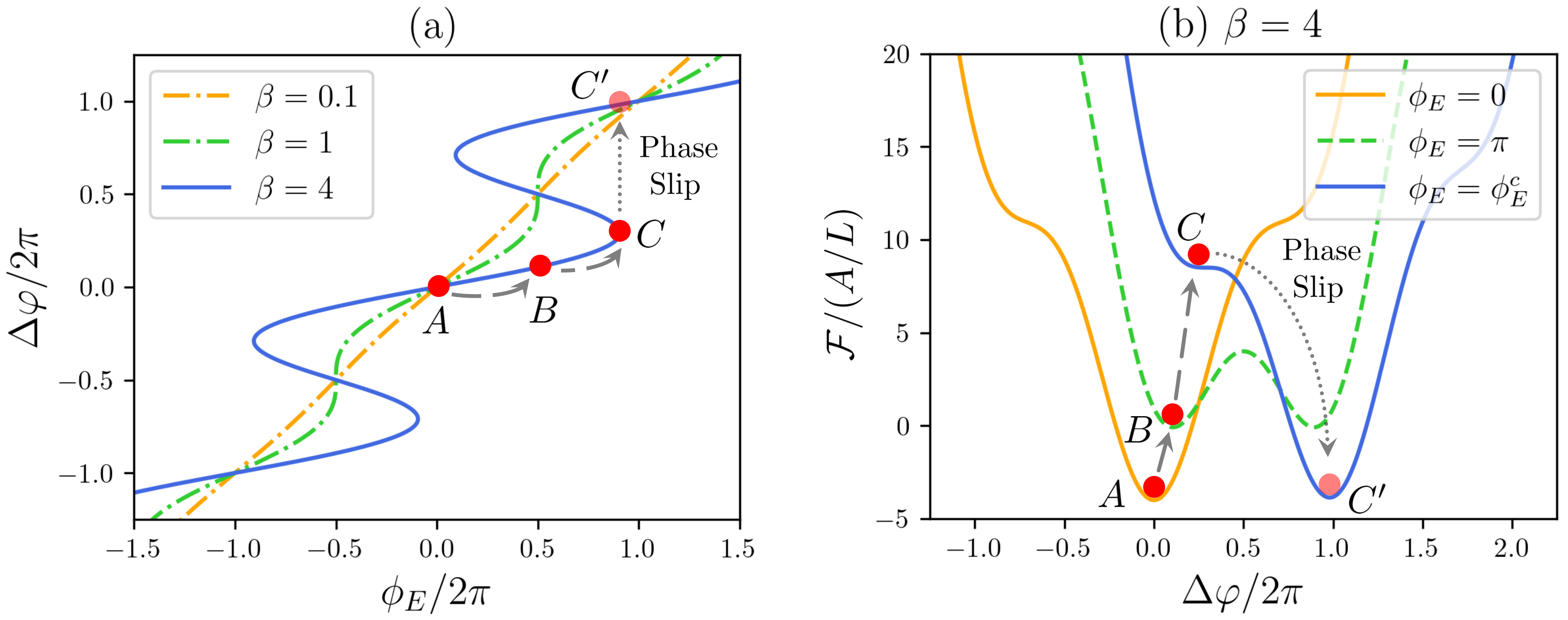
- At the *micromagnetic* level (via non-Abelian gauge transformation), this translates into a replacement of the order-parameter derivative in the exchange energy with the *chiral* derivative:

$$\partial_i \to \partial_i - \lambda(\mathbf{E} \times \mathbf{e}_i)\times \longrightarrow$$ Dzyaloshinski-Moriya interaction

- This also means that the spin current is proportional to an electric polarization: Produces electric field (analogously to electrical current in conventional SQUID producing magnetic field) - *dynamic vs geometric inductance*

Zhu, Kleinherbers, Levitov, and YT, *PRB* (2025)

Phase slips and electrical (Stark) frequency shifts



$$0 = \Delta\varphi - \phi_E + \beta \sin \Delta\varphi$$

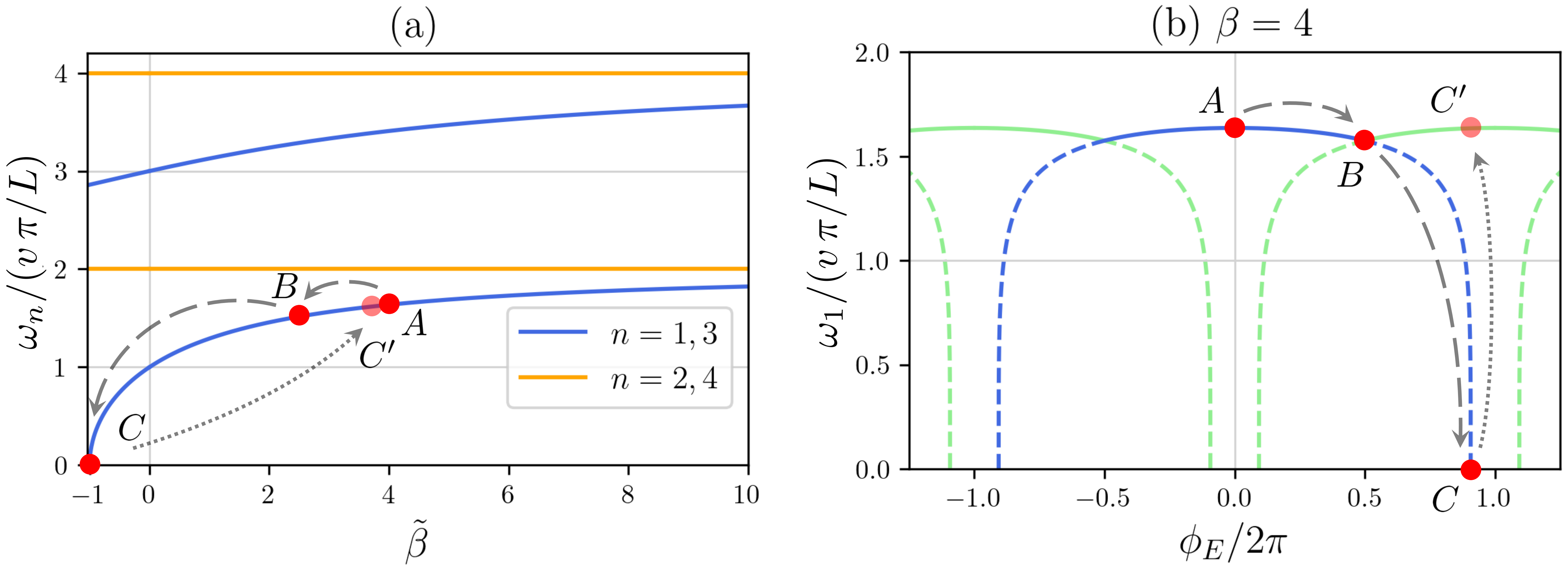
$$\boxed{\beta \equiv \frac{JL}{A}}$$

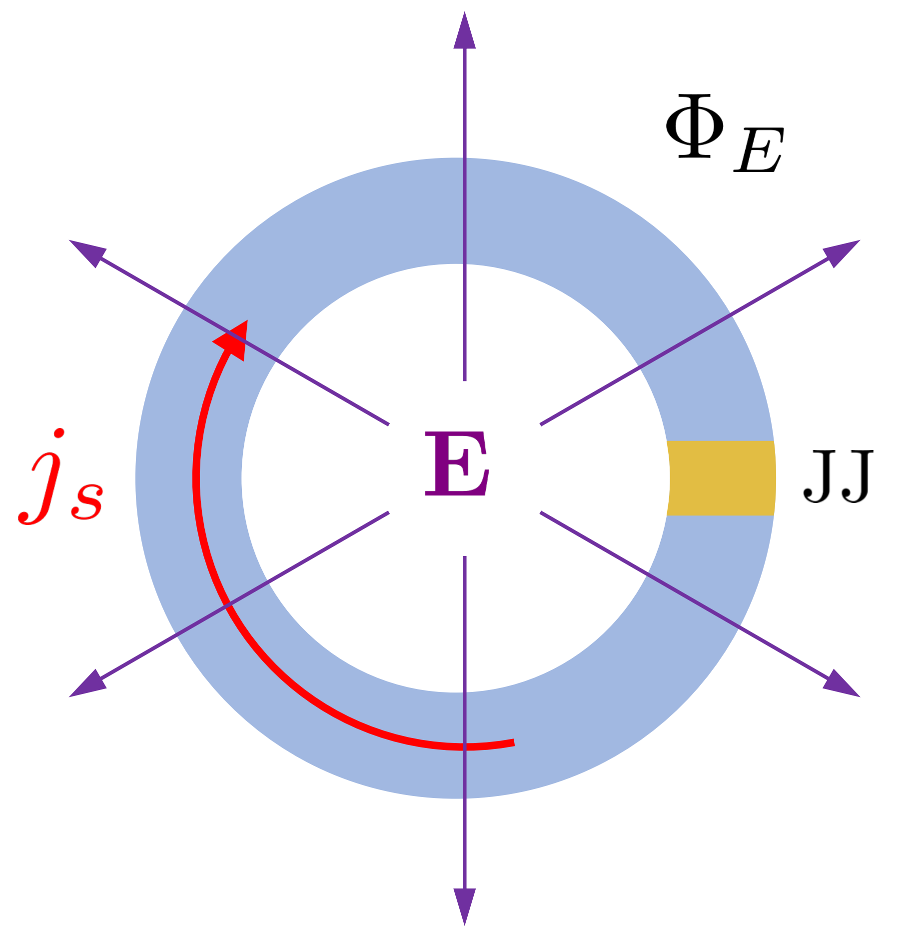
JJ coupling strength

$$\phi_E \equiv \int_0^L d\ell \, a_E = 2\pi \frac{\oint d\ell \, \mathbf{e}_n \cdot \mathbf{E}}{\Phi_E^0}$$

material-dependent flux quantum

Aharonov-Casher phase (AB dual)





The state just before the phase slip (C) is most sensitive to the phase change

Zhu, Kleinherbers, Levitov, and YT, *PRB* (2025)

We can apply these ideas to unconventional superconductors without weak links

Topological hydrodynamics in ferromagnetic superconductors

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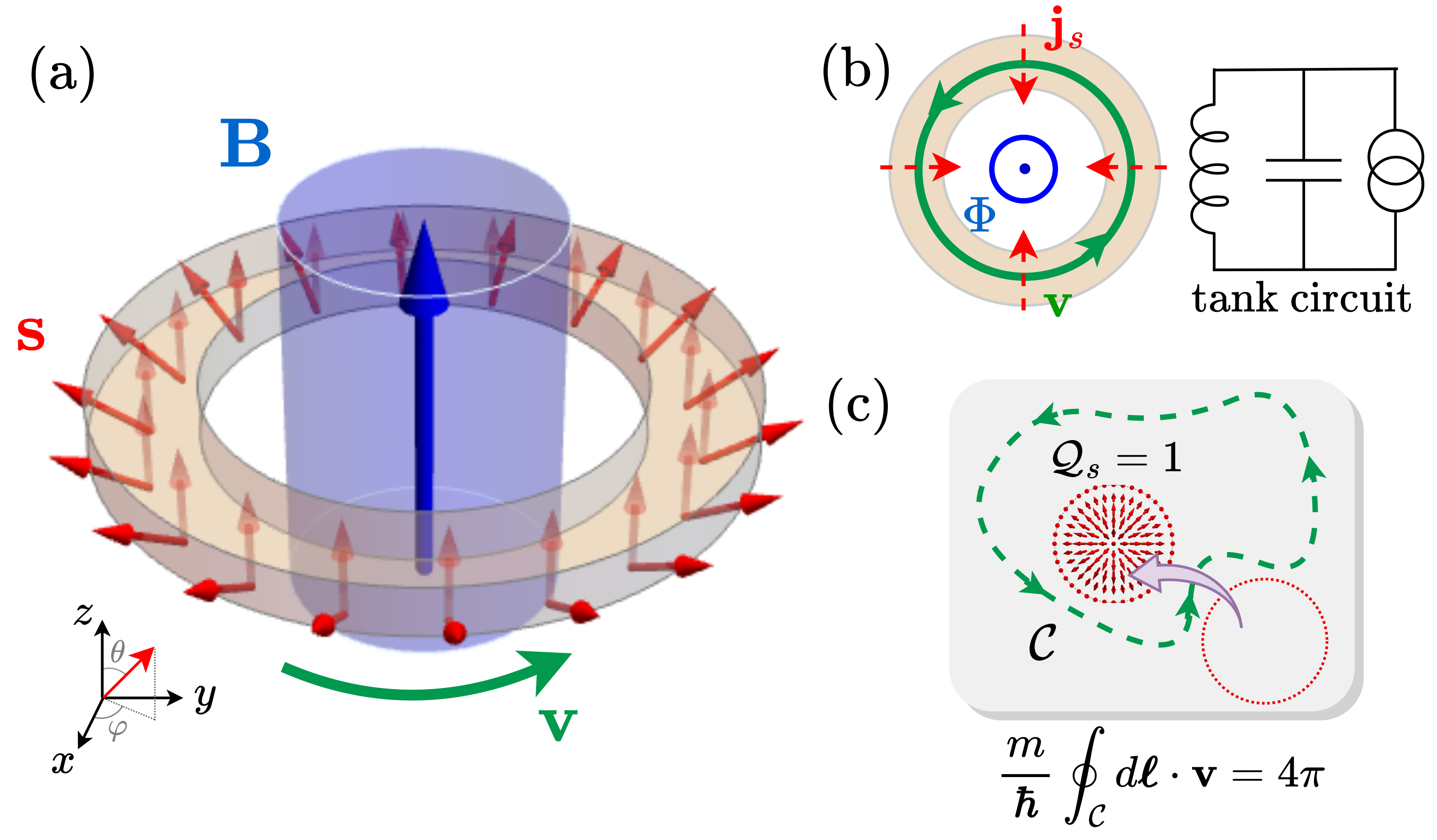
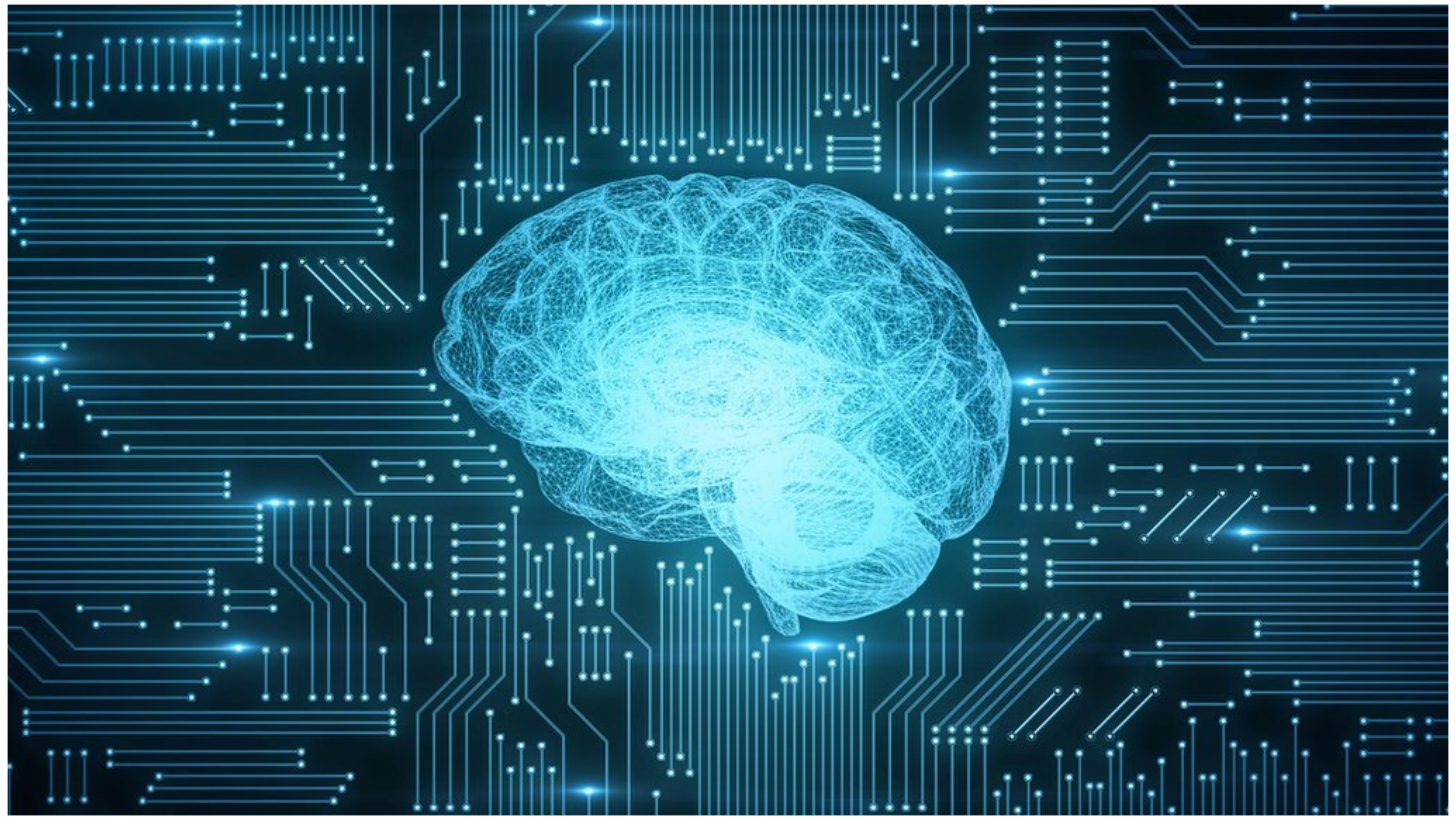
(Dated: April 15, 2025)

We present the hydrodynamic equations of motion for spin-triplet superconductivity, in which the superconductor can be understood as a charged ferrofluid. Due to the structure of the underlying **d**-vector order parameter, the dynamics of the superconductor fulfill a topological constraint linking the fluid vorticity to the magnetic skyrmion density. To probe this interplay of charge and spin dynamics, we propose a blueprint for a spin-triplet superconducting quantum interference device (SQUID), which functions without a Josephson weak link. The triplet SQUID undergoes *nonsingular* 4π phase slips, in which current relaxation is facilitated by spin dynamics that trace out a magnetic skyrmion texture. Inductively coupling the device to a tank circuit and probing the nonlinear supercurrent response via Oersted field measurements could provide an experimental signature of ferromagnetic spin-triplet superconductivity.



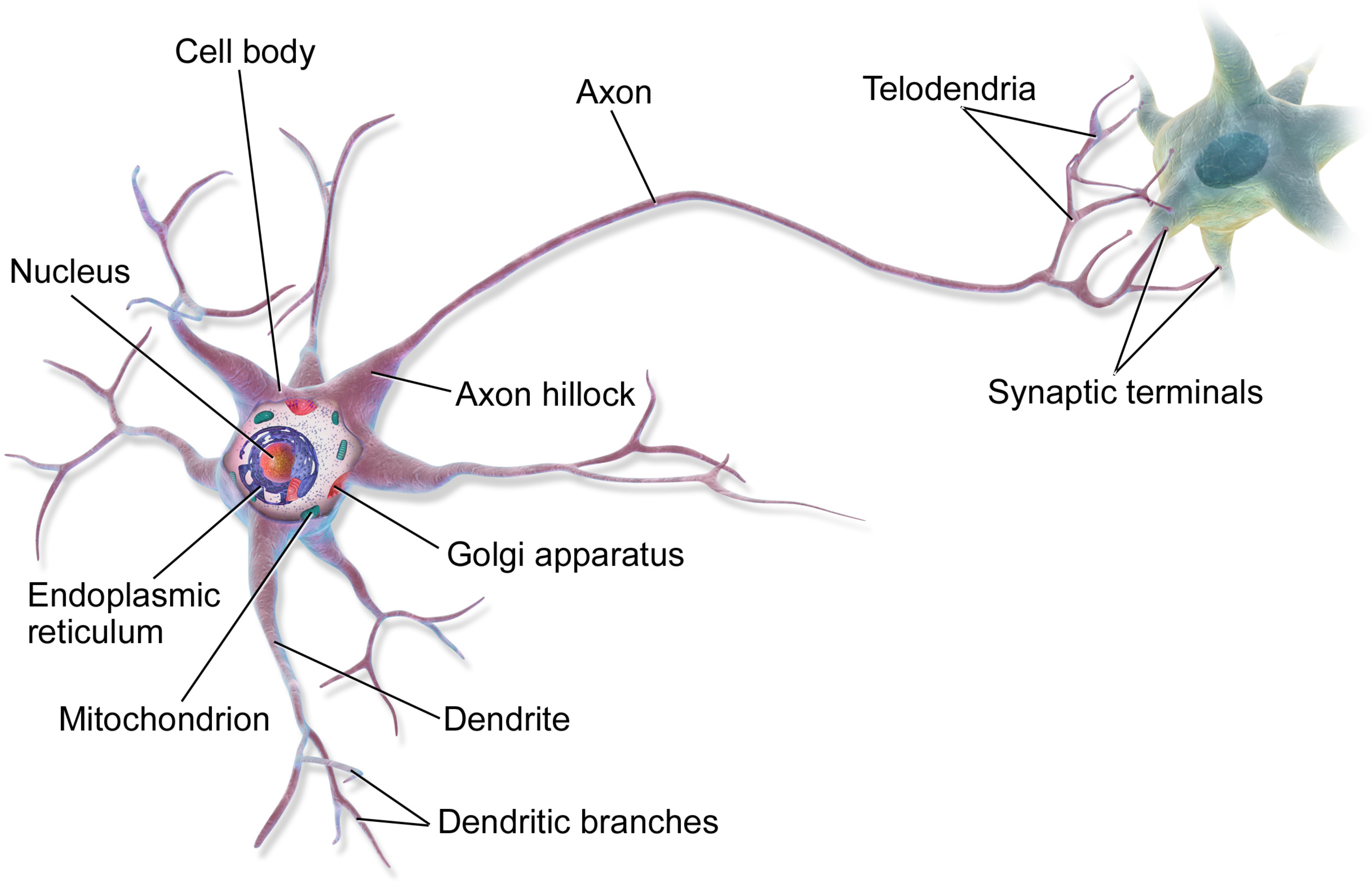
- We have recently extended these ideas to unconventional (*triplet*) superconductors, where the SO(3) order parameter topology allows for nonsingular 4π phase slips
- This enables (conventional) *rf SQUID* functionality without a need for Josephson junctions
- These phase slips are associated with radial skyrmion passage (*Mermin-Ho relation*)
- A route to novel devices and *signatures* of unconventional superconductivity

Potential in "topological neuromorphics" (AI hardware)?

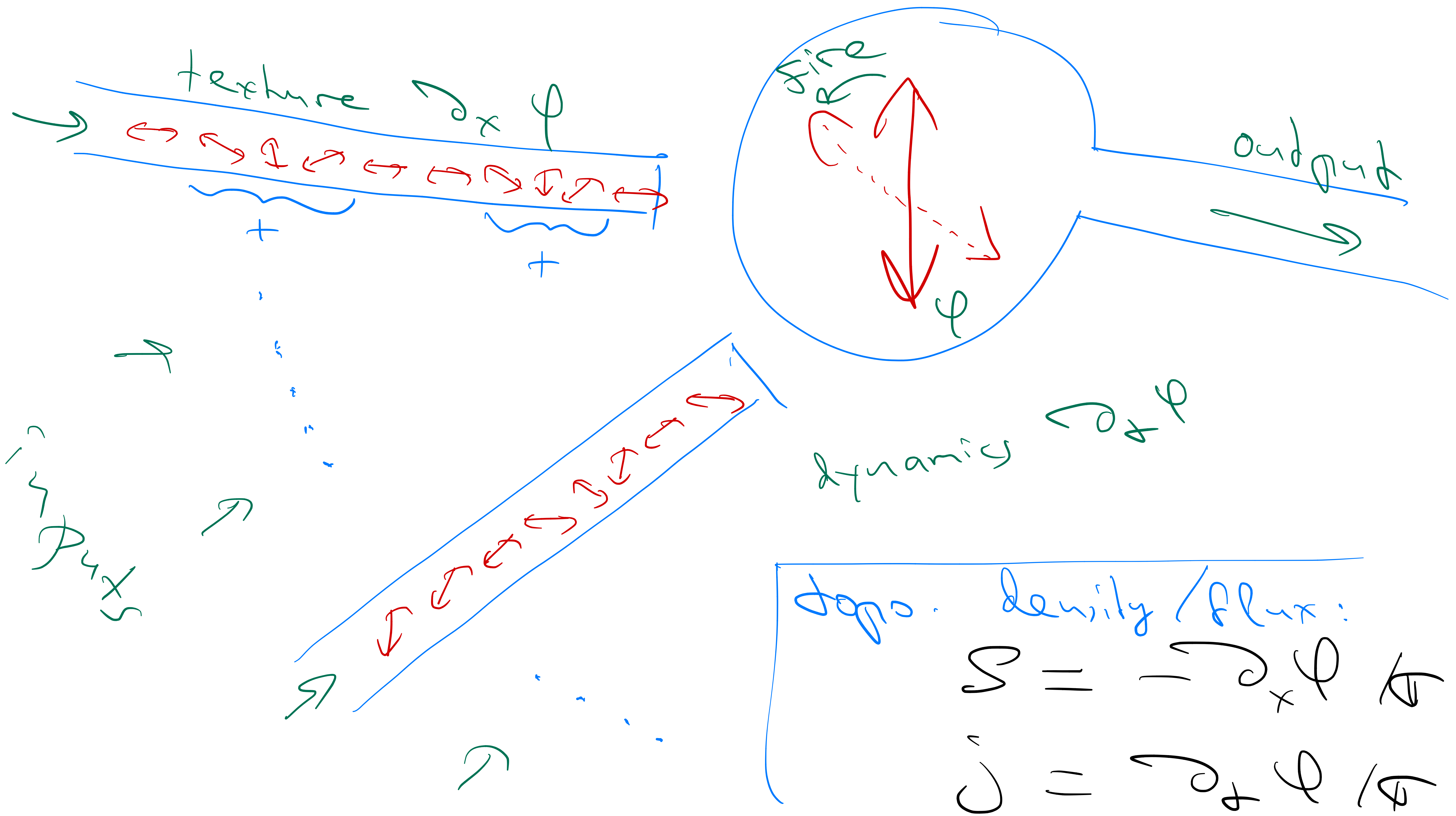


- Continued progress in machine (especially *unsupervised*) learning and artificial intelligence may benefit from dedicated *analog hardware* that is designed to mimic brain-like behaviors
- *Topological texture* transport and dynamics in magnetic insulators appears to provide all the conceptual building blocks, including *solitonic signal propagation*, *nonlinear signal integration*, and *plasticity*
- This is further complemented by built-in *energy storage* capability

Biological neuron



Spatiotemporal winding textures can mimic synapses and synaptic transmission

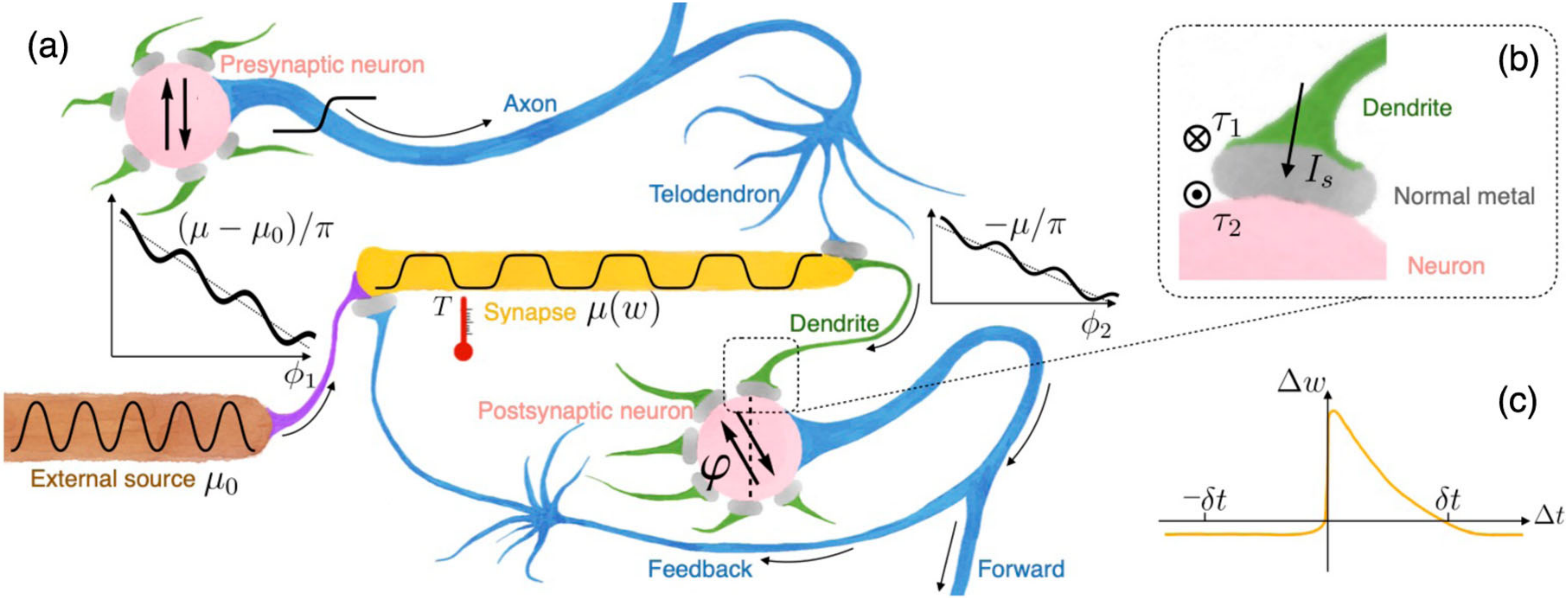


topo. density / flux:

$$\rho = -\partial_x \varphi / 2\pi$$

$$j = \partial_t \varphi / 2\pi$$

Our proposed (dream) schematic:



Zhang and YT, *PRL* (2020)

Consider a Lorentz-boosted soliton exiting a magnetic wire (to enter a "synapse"):

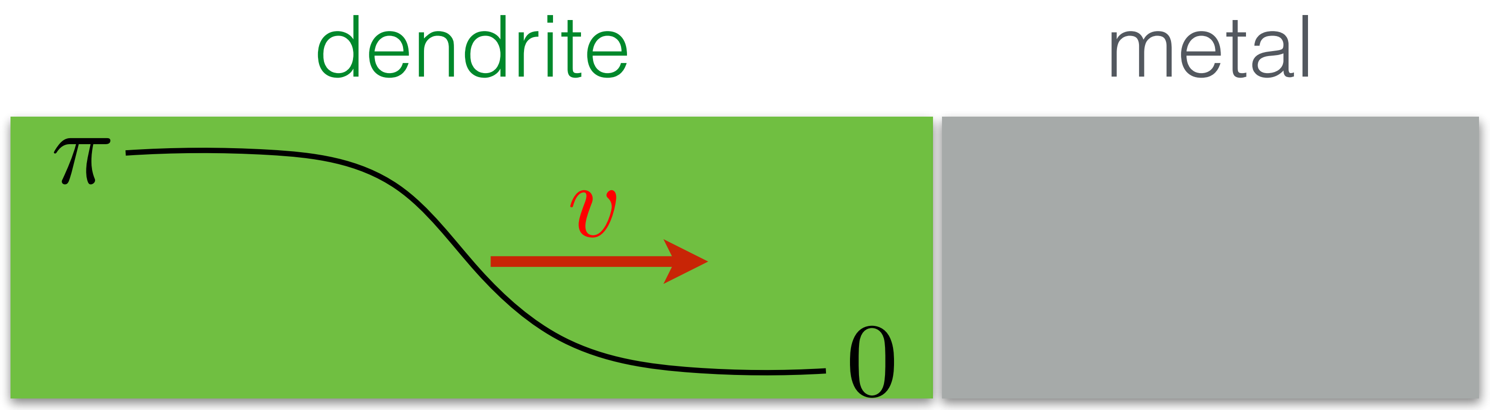
- 1D winding dynamics obeys damped sine-Gordon equation:

$$\frac{\partial_t^2 \varphi}{c^2} - \partial_x^2 \varphi + \frac{\sin 2\varphi}{2\lambda^2} + \alpha \partial_t \varphi = 0$$

- For weak dissipation α, elementary topological charges move as relativistic solitons:

$$\varphi = 2 \tan^{-1} \left[\exp \left(-\frac{x - vt}{\lambda \sqrt{1 - (v/c)^2}} \right) \right]$$

- Depending on the boundary conditions, these solitons can get either fully transmitted or fully reflected at a wire's edge:



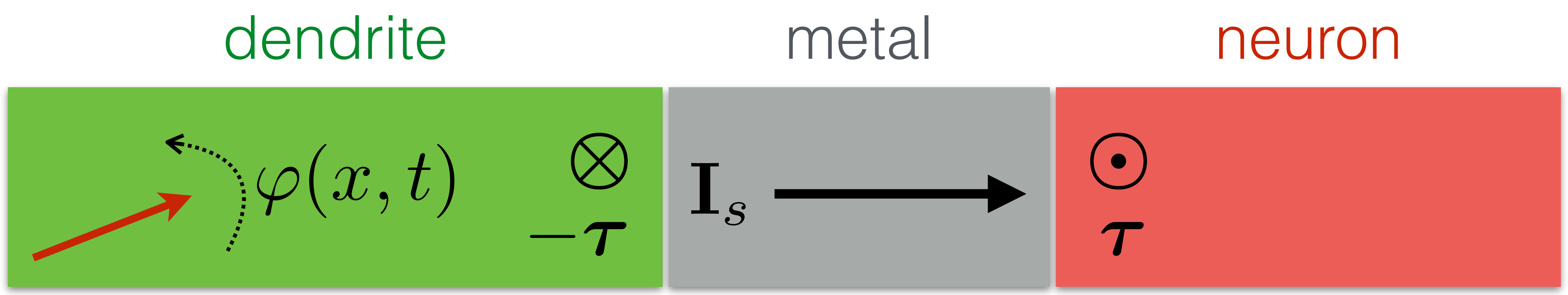
$N = 1$

$$\Delta N = \Delta \varphi_{\text{boundary}} / \pi \in \mathbb{Z}$$

change of the total topological charge in the wire

Zhang and YT, *PRL* (2020)

Adding spin-transfer and spin-pumping torques at metallic interfaces:



(weak link: *synapse*)

- A metallic interconnect between a "dendrite" and a "neuron" mediates coupling by adiabatic spin pumping and spin-transfer torque (supposing the electron diffusion is much faster than the magnetic dynamics)

$$\tau = I_s = \frac{\hbar g^{\uparrow\downarrow}}{8\pi}\partial_t\varphi$$

- The corresponding exchange magnetic boundary condition (spin continuity) is:

$$-\tau = \mathcal{A}\partial_x\varphi$$

- This gives "impedance matching" condition for perfect soliton transmission into the metal link:

$$v = \frac{8\pi\mathcal{A}}{\hbar g^{\uparrow\downarrow}}$$

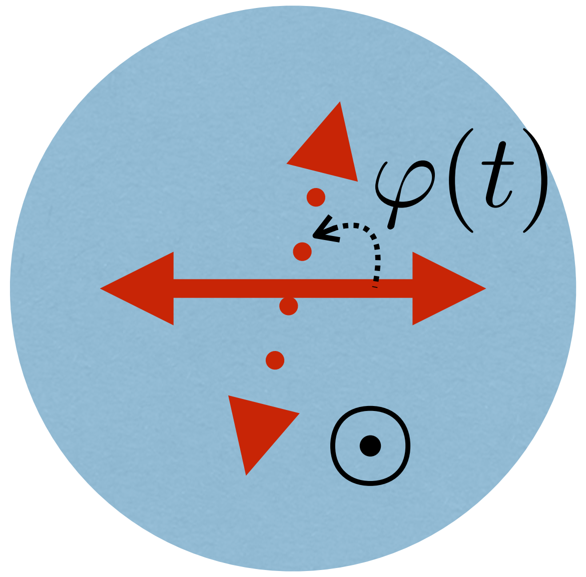
YT, Brataas, Bauer, and Halperin, *RMP* (2005)

Leaky integrate-and-fire

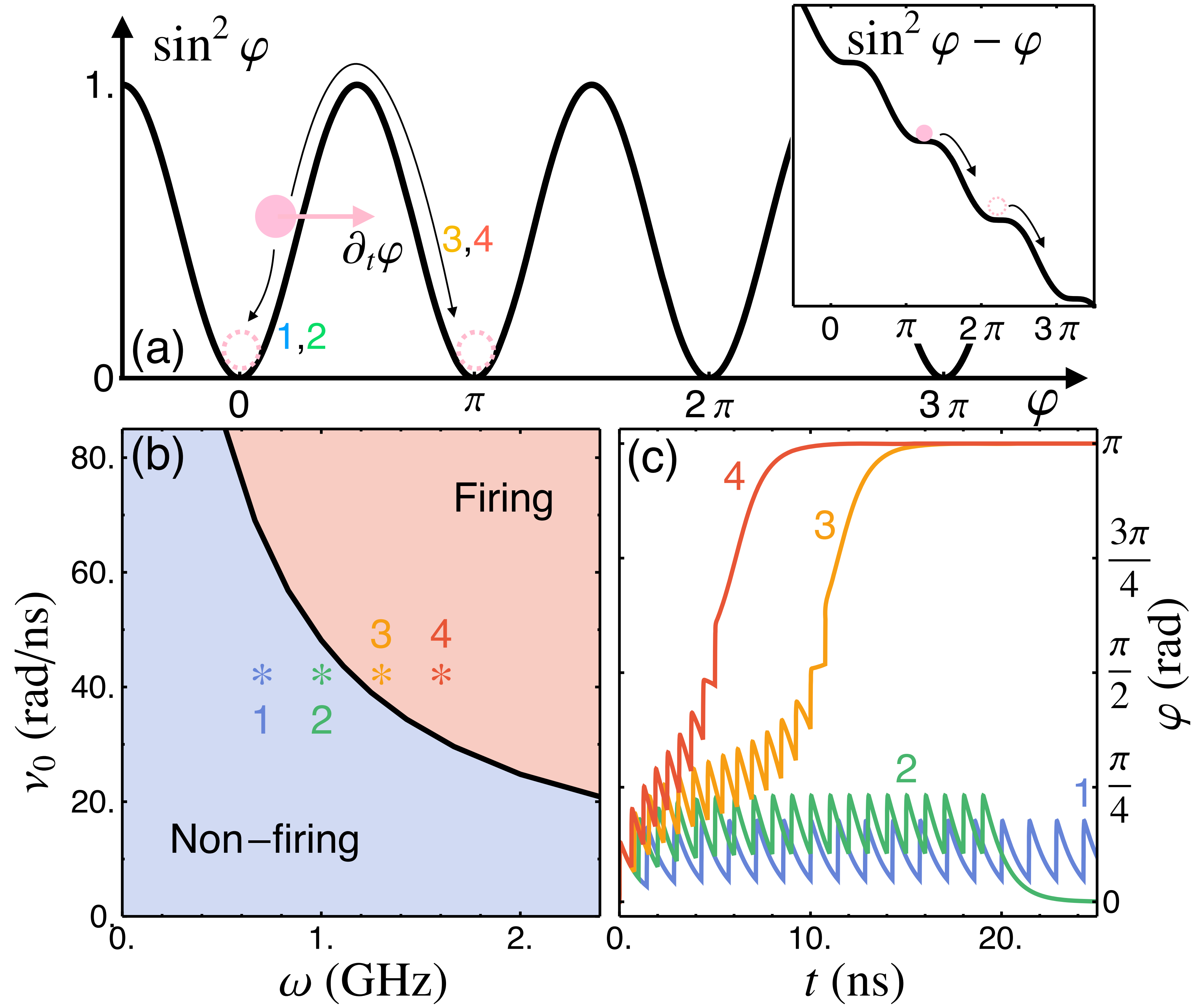
- Monodomain "neuron" dynamics maps on the forced motion of a damped massive particle in a washboard potential:

$$\partial_t\varphi = \frac{S}{\chi}\,, \quad \text{and} \quad \partial_t S = -\frac{K\sin 2\varphi}{2} - \alpha\partial_t\varphi + \tau(t)$$

inertia washboard damping



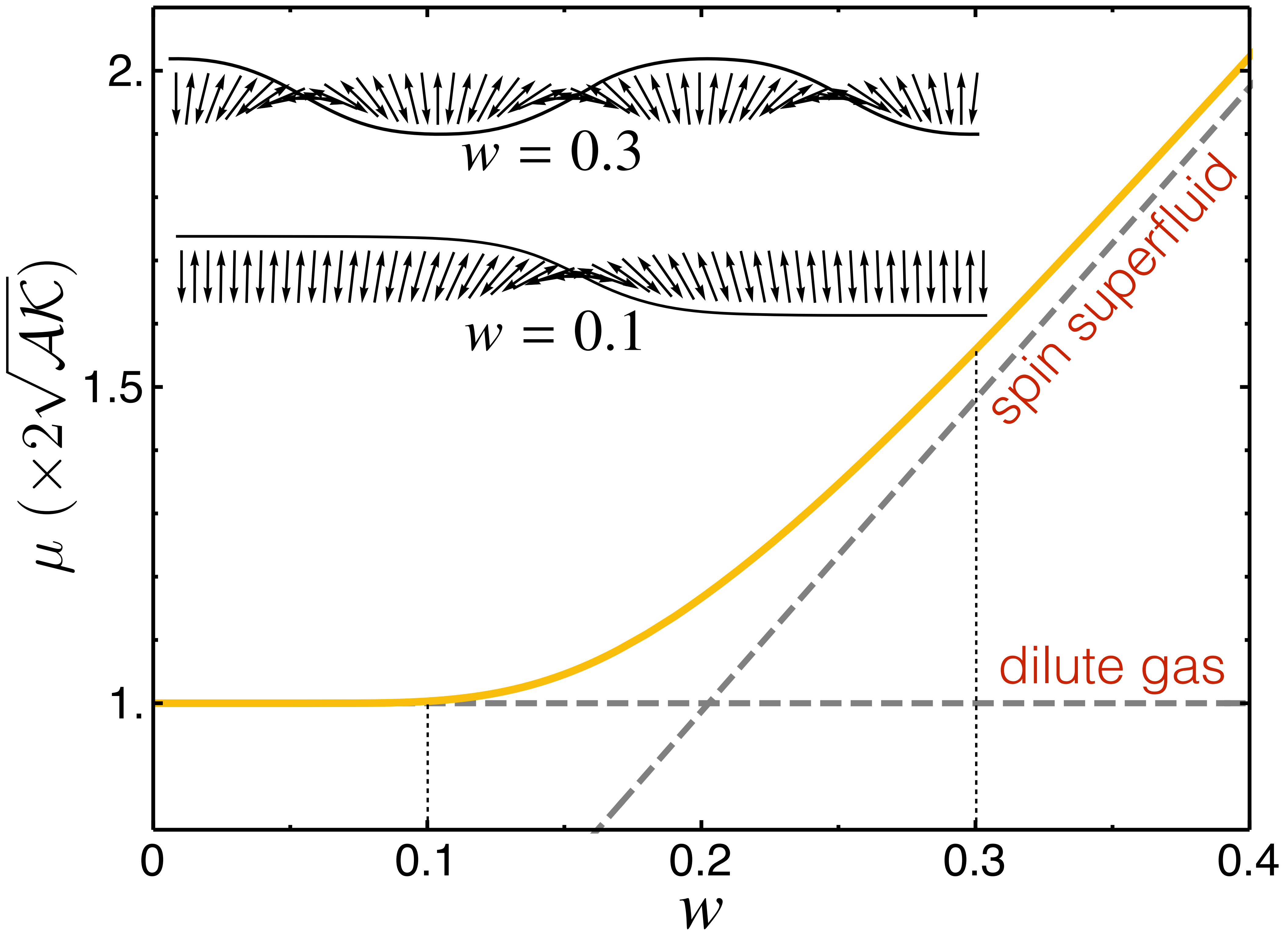
$S(t)$
canonical momentum



Zhang and YT, *PRL* (2020)

Synapse as a thermodynamic soliton liquid

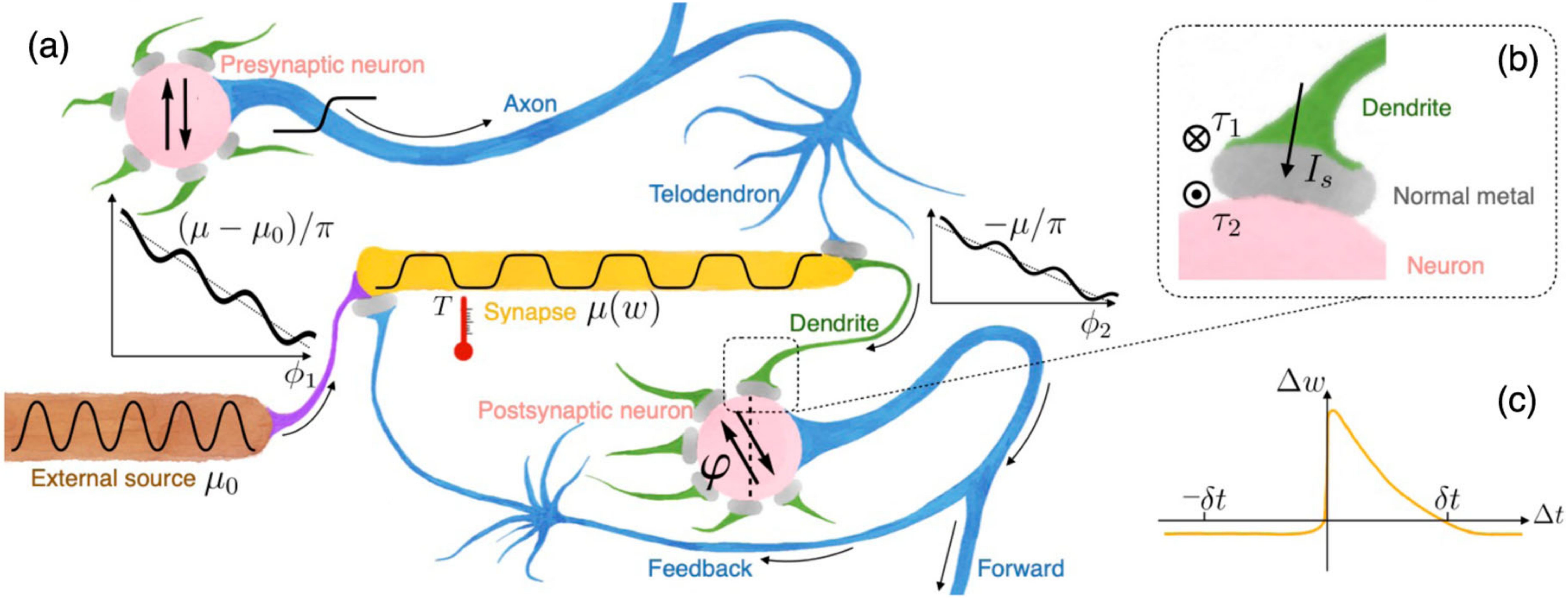
- The modality of a sine-Gordon model that realizes a *synapse* is based on a *thermodynamic* description with a *chemical potential* μ and temperature T associated with the topological charge distribution



synaptic weight: topological charge density

Zhang and YT, *PRL* (2020)

Spike-timing dependent plasticity



- The synaptic weight w gets self-updated as a function of relative timing Δt between the post- and pre-synaptic signals

- In order to raise the weight, a free energy is loaded from an external source (where the topological charge is maintained at a fixed chemical potential μ_0)

- The latter can also generically provide an amplification/generation of the winding dynamics whenever needed

Zhang and YT, *PRL* (2020)

(Proof-of-principle) applications validate the concepts

- **Nonchemical energy storage**
    - *Magnetism can provide both stability and sizable free energy, per atomic bond*
- **Sensing**
    - *Motivated by SQUID, we propose a dual spin-superfluid QUID for electric-field sensing*
- **AI hardware**
    - *Topological degrees of freedom naturally provide nonlinearities, transport/communication, information storage/learning, and interface with standard electronics*