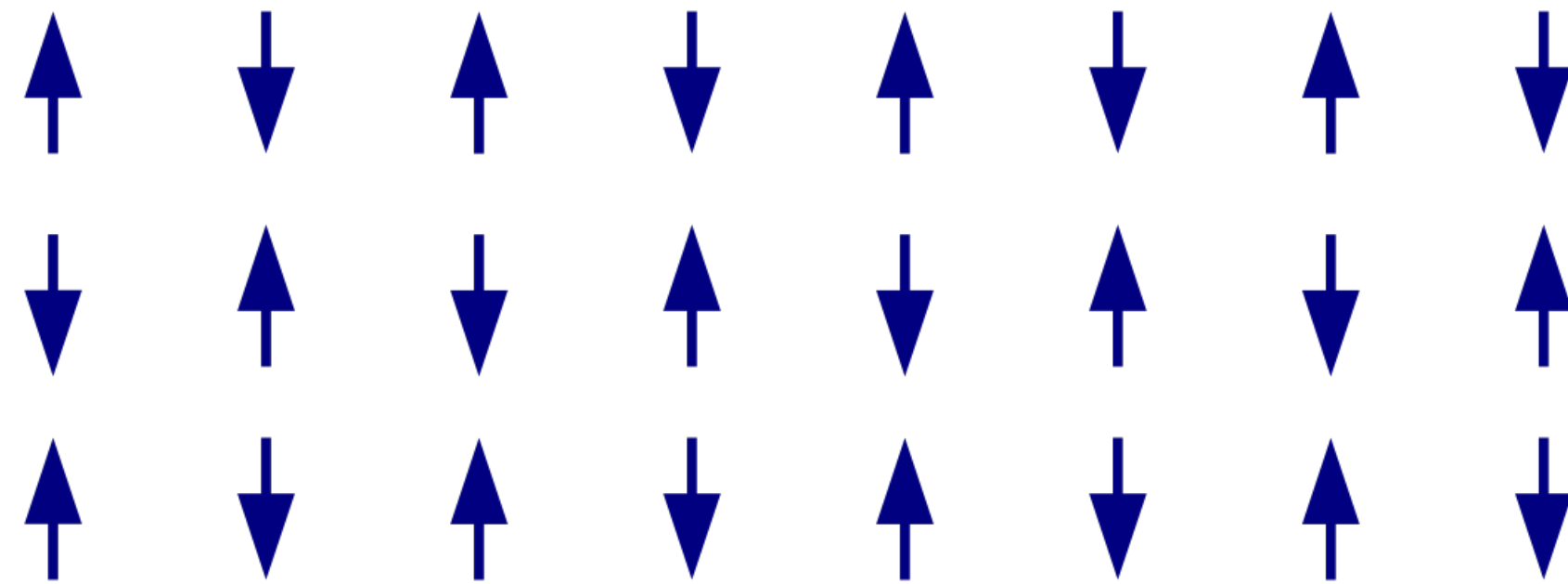


Novel transport: Engendered by topology, enabled by symmetry

Conceptual construction and basic physics: From *homotopy* to *conservation laws* and *nonequilibrium thermodynamics*



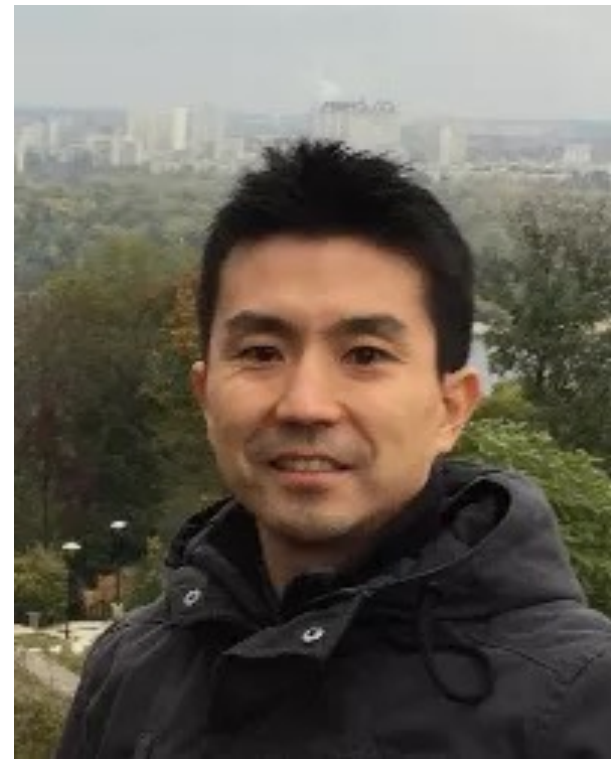
Yaroslav Tserkovnyak



People



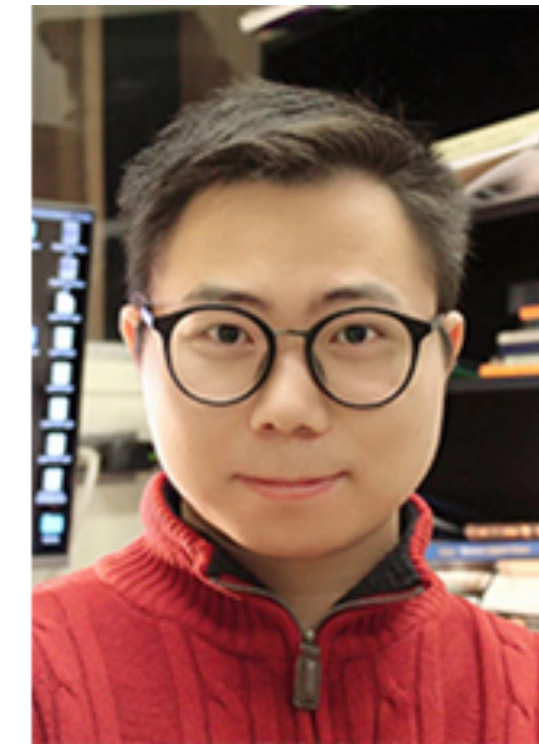
Bert Halperin (*Harvard*)



So Takei (*CUNY*)



Se Kwon Kim (*KAIST*)



Ji Zou (*KFUPM*)



Chau Dao (*Basel*)



Amir Yacoby (*Harvard*)



Hector Ochoa (*Columbia*)



Ricardo Zarzuela (*Mainz*)

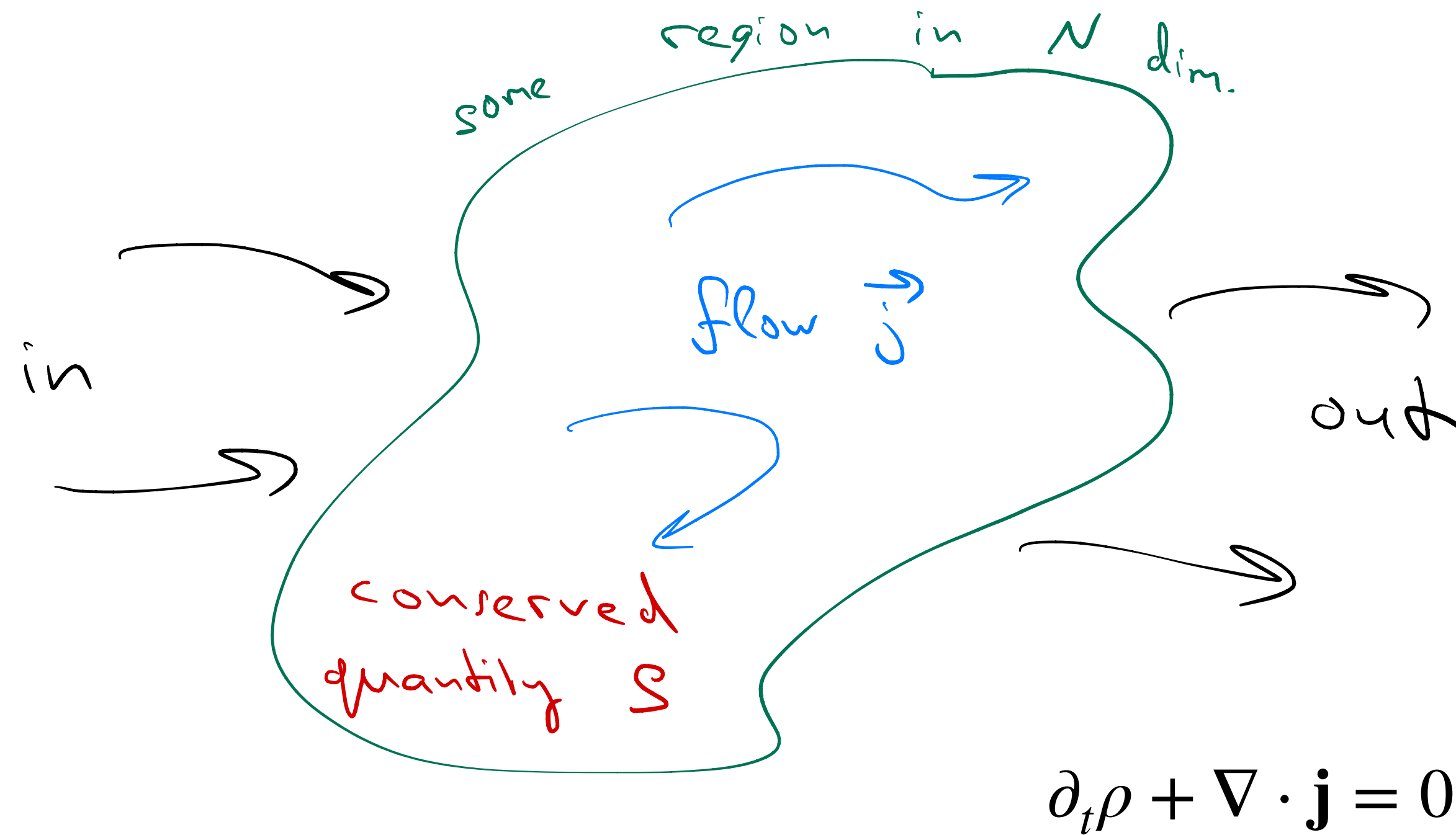


Suzy Zhang (*Okinawa*)



Eric Kleinherbers (*UCLA*)

Conservation laws engender transport phenomena



Emmy Noether

Two distinct routes to obtain conserved quantity $Q = \int dV \rho$:

- **generator of symmetry:** *gauge* (particle number), *space-time translational* (linear momentum and energy), *rotational* (angular momentum) etc.
- **topological charge:** winding, vector chirality (vorticity), scalar chirality (skyrmion) etc.

Central role of transport phenomena in:

- **Fundamental classification of materials**
 - *Metals, insulators, semiconductors, superconductors etc.*
- **Integral part of low-energy theories**
 - *In and out of equilibrium: Collective modes, diffusion, slow relaxation*
- **Applications** (see 2nd talk)
 - *Material probes, electronics, spintronics, sensing, energy storage*

Transport of topological charges remains mostly elusive

...except for superfluidity/superconductivity:



The robustness of the superfluidity stems from the *topological character of the superflow*:

Free energy $F[\rho_c(\mathbf{r}), \rho_w(\mathbf{r})]$ depends on the charge, ρ_c , and *phase-winding*, ρ_w , densities:
(electric charge) (topological charge)

$$\partial_t \rho_c + \partial_x j_c = 0$$

$$j_c \propto \rho_w \equiv \partial_x \varphi$$

$$\partial_t \rho_w + \partial_x j_w = 0$$

$$j_w \equiv -\partial_t \varphi$$

Perspective: (Beyond) spin transport in insulators

Yaroslav Tserkovnyak

Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

(Received 29 August 2018; accepted 3 November 2018; published online 21 November 2018)

Insulating materials with dynamical spin degrees of freedom have recently emerged as viable conduits for spin flows. Transport phenomena harbored therein are, however, turning out to be much richer than initially envisioned. In particular, the topological properties of the collective *order-parameter textures* can give rise to *conservation laws* that are not based on any specific symmetries. The emergent continuity relations are thus robust against structural imperfections and anisotropies, which would be detrimental to the conventional spin currents (that rely on approximate spin-rotational symmetries). The underlying fluxes thus supersede the notion of spin flow in insulators, setting the stage for nonequilibrium phenomena termed *topological hydrodynamics*. Here, we outline our current understanding of the essential ingredients, based on the *energetics* of the electrically-controlled injection of topological flows through interfaces, along with a reciprocal signal generation by the outflow of the conserved quantity. We will focus on two examples for the latter: winding dynamics in one-dimensional systems, which supplants *spin superfluidity* of axially-symmetric easy-plane magnets, and *skyrmion* dynamics in two-dimensional Heisenberg-type magnets. These examples will illustrate the essential common aspects of topological flows and hint on generic strategies for their generation and detection in spintronic systems. Generalizations to other dimensions and types of order-parameter spaces will also be briefly discussed. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5054123>

microscopics:

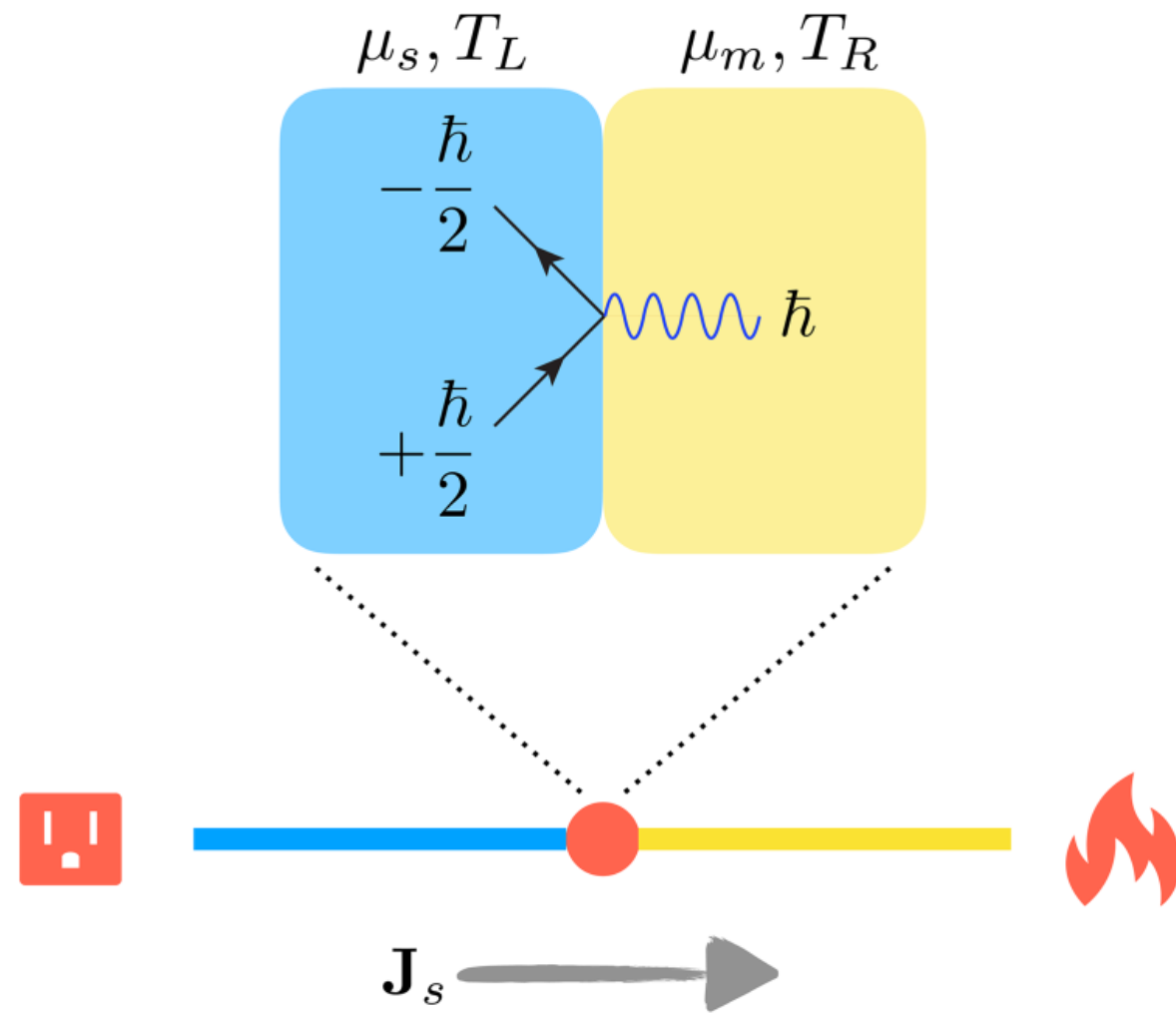


FIG. 1. A schematic of an elementary spin-transport node between a non-magnetic metal (left) and a magnetic insulator (right). Electrons can flip their spin at the interface, while transmitting (or absorbing) a magnon. The spin current J_s is driven by the thermodynamic bias of $(\mu_s - \mu_m)$, in spin sector, and $(T_L - T_R)$, in heat sector. Such a bias across the interfacial node can be established in response to thermoelectric controls of a larger circuit, in a self-consistent steady state.^{17,18}

coarse-grained torques:

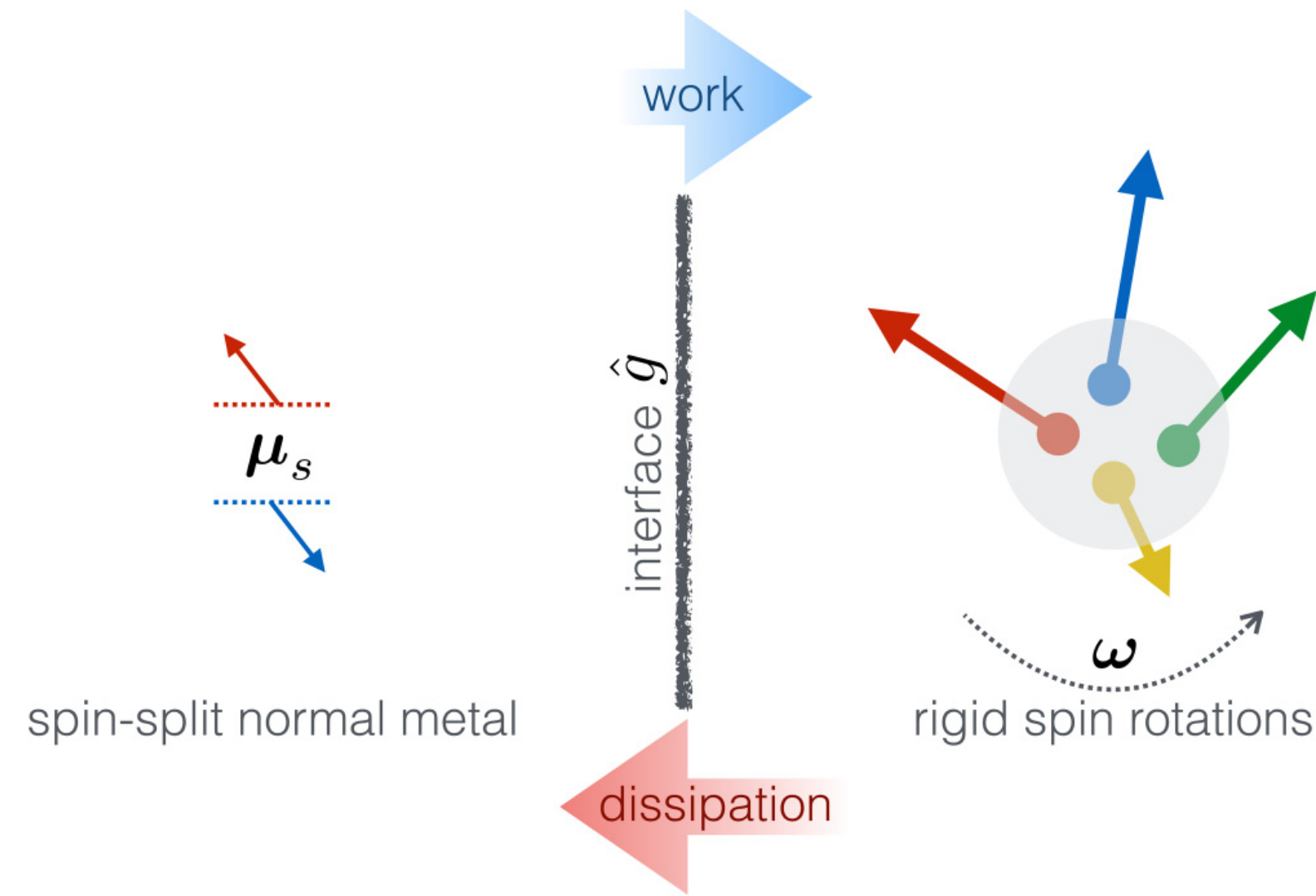


FIG. 2. A schematic of a noncollinear spin system (right) in contact with a metallic spin reservoir (left). The nonequilibrium spin state of the metal is parametrized by the (vectorial) spin accumulation μ_s . The magnet, whose spin arrangement is determined by some isotropic exchange Hamiltonian, is described, near the interface, by uniform (and essentially rigid) rotations of all spins. Its instantaneous nonequilibrium state is thus characterized by the (vectorial) frequency of SO(3) rotation ω . The 3×3 matrix $\hat{g}$ generalizes the concept of the spin-mixing conductance g pertinent to the collinear case. Adapted from Y. Tserkovnyak and H. Ochoa, Phys. Rev. B **96**, 100402(R) (2017). Copyright 2017 American Physical Society.

coherent-texture transport:

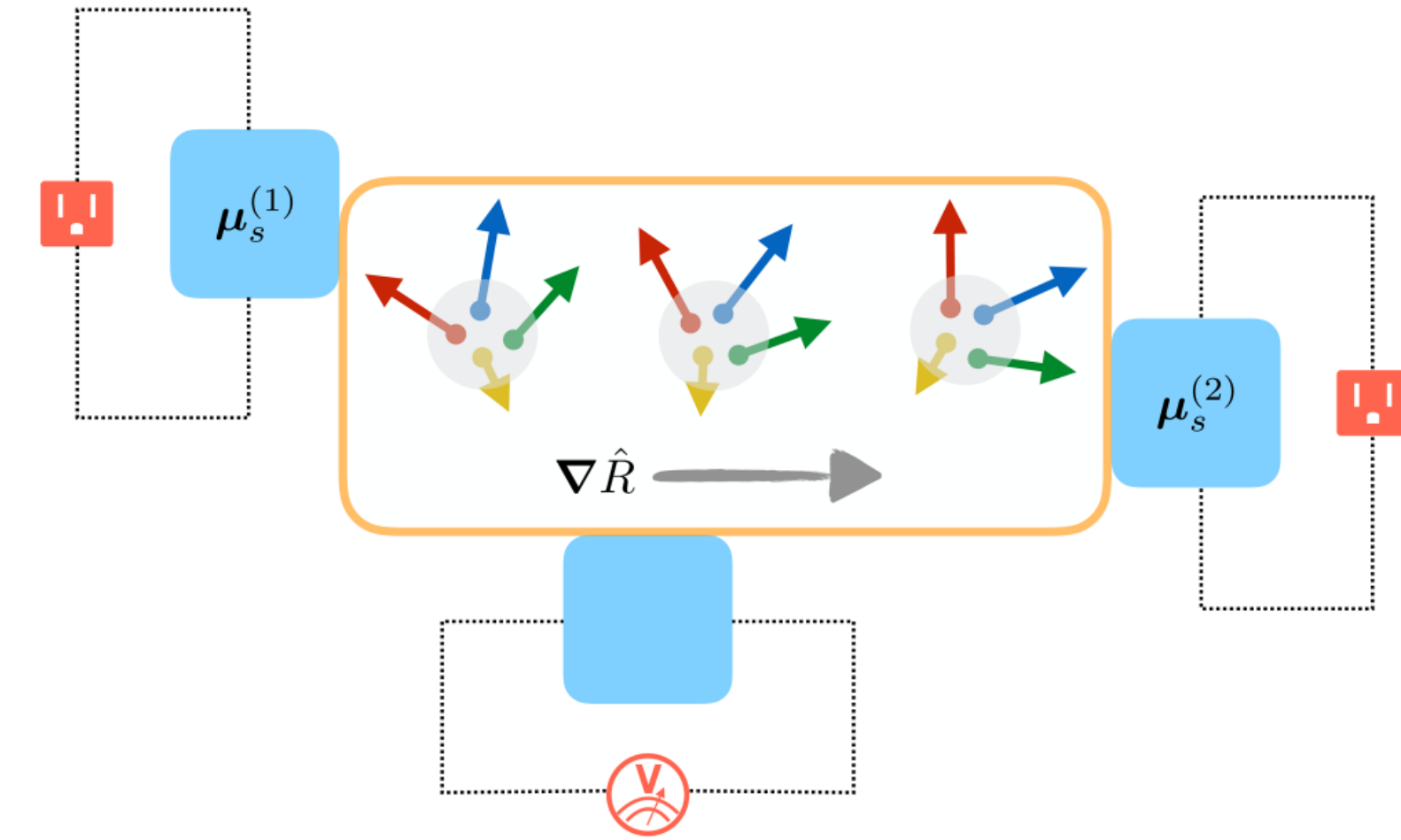
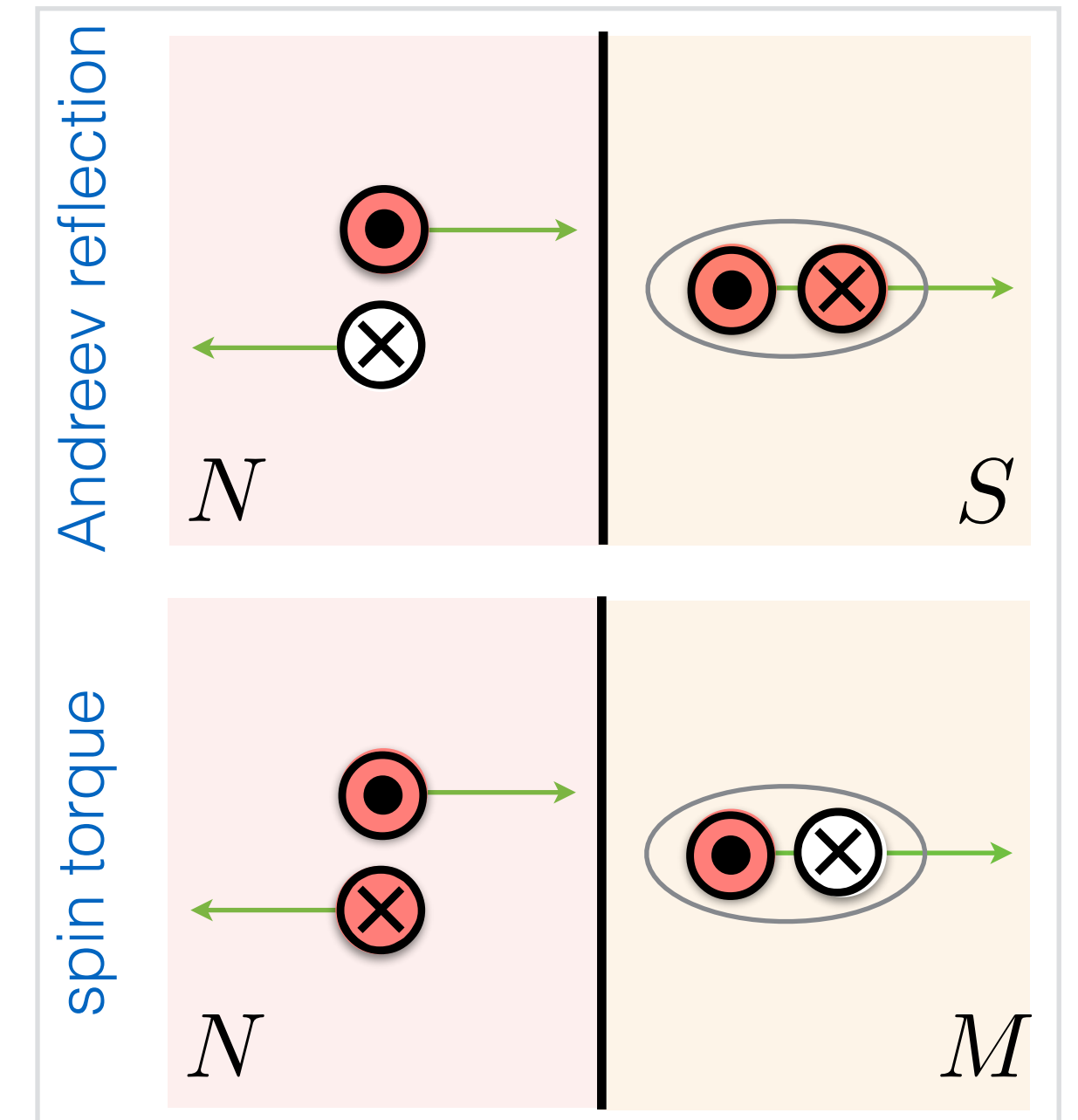
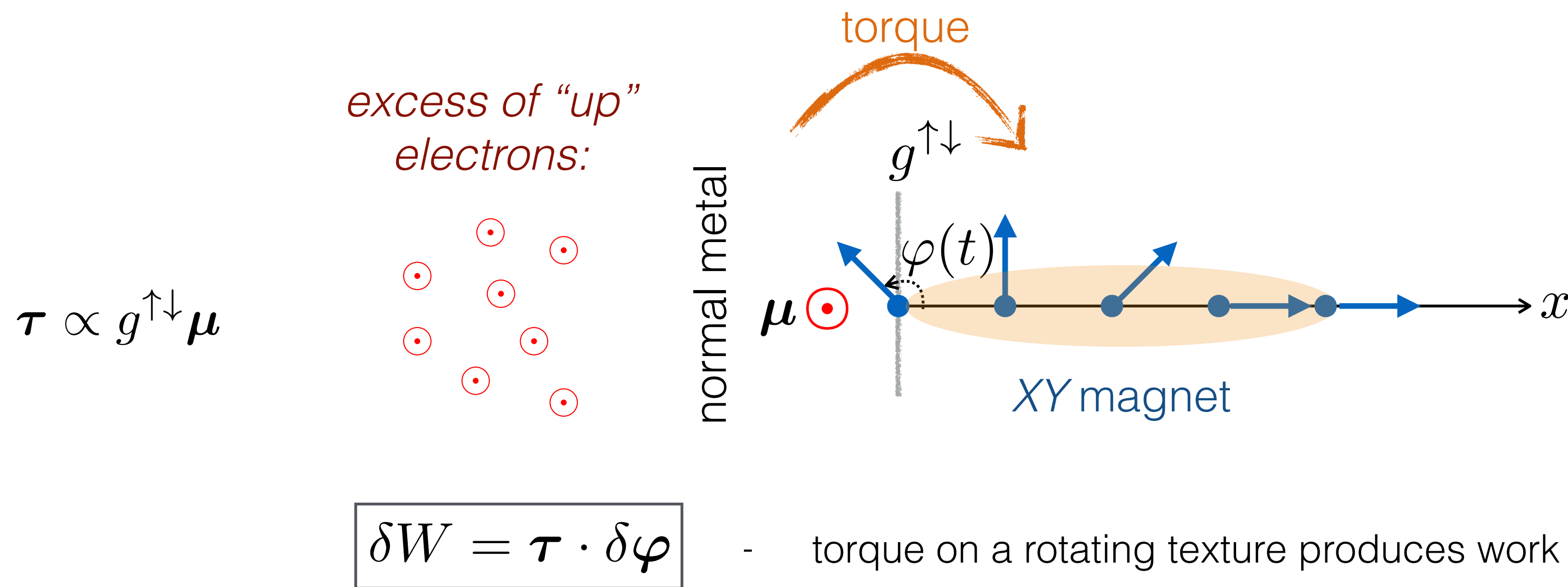
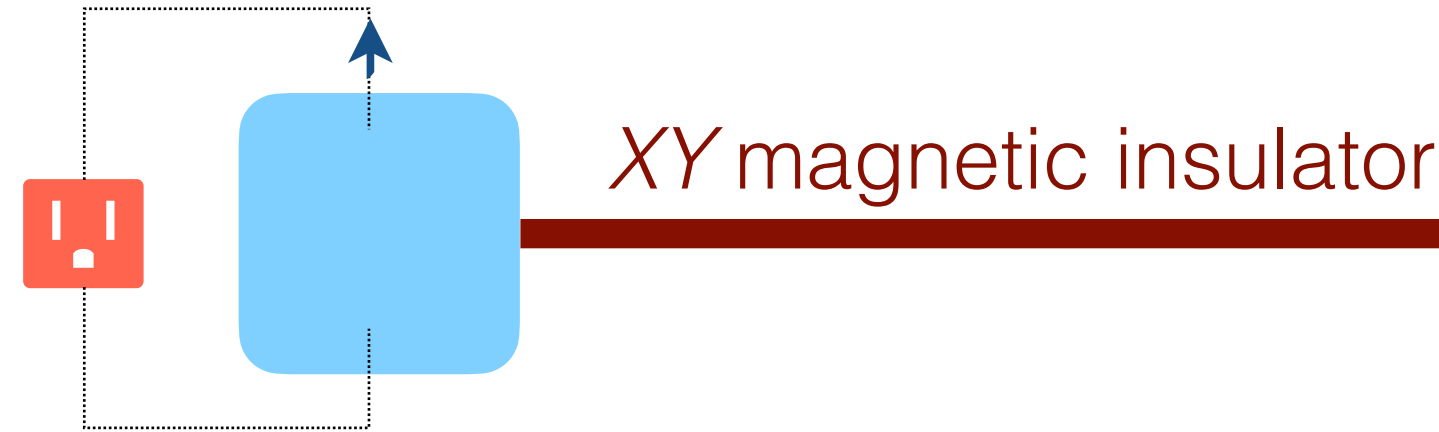


FIG. 3. A general spin circuit, in which the input terminals labeled by $i = 1, 2$, establish local spin biases $\mu_s^{(i)}$, which drive spin dynamics in the material or system of interest. The readout terminal (bottom) performs a measurement of the resultant dynamic steady state via reciprocal means. For example, if the input is accomplished by the spin Hall effect, by applying electrical currents that induce spin accumulations (8), the output can be achieved by measuring the inverse spin Hall voltage.¹⁰ The instantaneous state of the driven system can be described, e.g., by the position-dependent rotation matrix $\hat{R}(\mathbf{r})$, supposing a rigid local order.

Magnetic planar winding serves as a conceptual blueprint



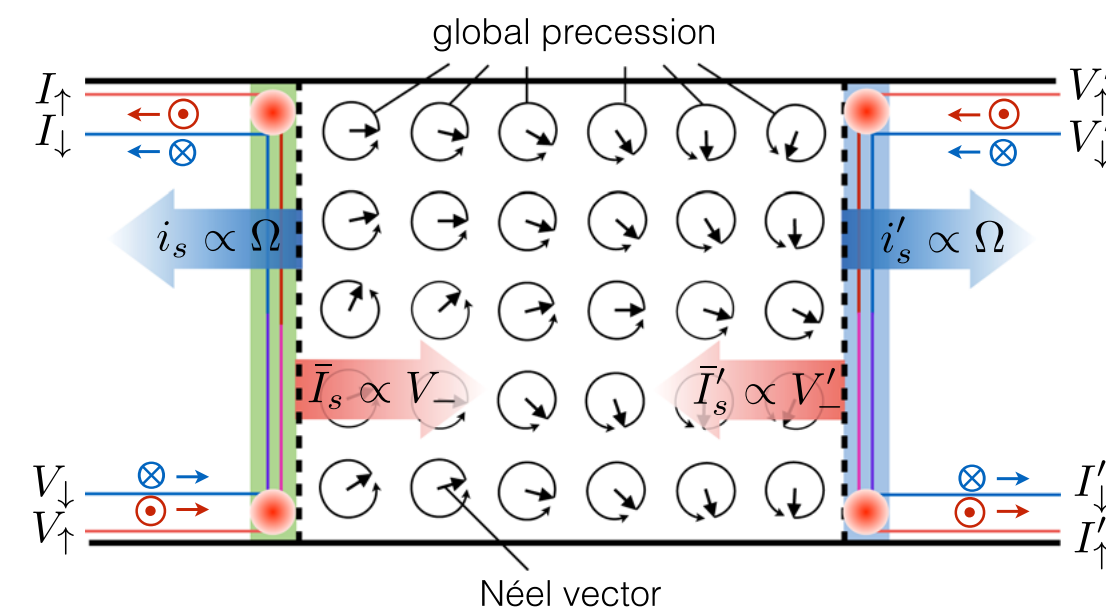
- out-of-plane spin accumulation in the adjacent normal metal triggers injection of the winding density $\rho_w \equiv \partial_x \varphi$ into the easy-plane magnet
- spin-z rotational symmetry is analogous to U(1) gauge symmetry; even in the absence of this symmetry, such that the genuine (idealized) spin superfluidity breaks down, the winding hydrodynamics survives: smooth winding merely fragments into (chiral) solitonic domain walls

This gives us an opening for broader unconventional transport studies

PRL **116**, 216801 (2016) PHYSICAL REVIEW LETTERS week ending 27 MAY 2016

Spin Superfluidity in the $\nu=0$ Quantum Hall State of Graphene

So Takei,^{1,2} Amir Yacoby,³ Bertrand I. Halperin,³ and Yaroslav Tserkovnyak²
¹Department of Physics, Queens College of the City University of New York, Queens, New York 11367, USA
²Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA
³Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

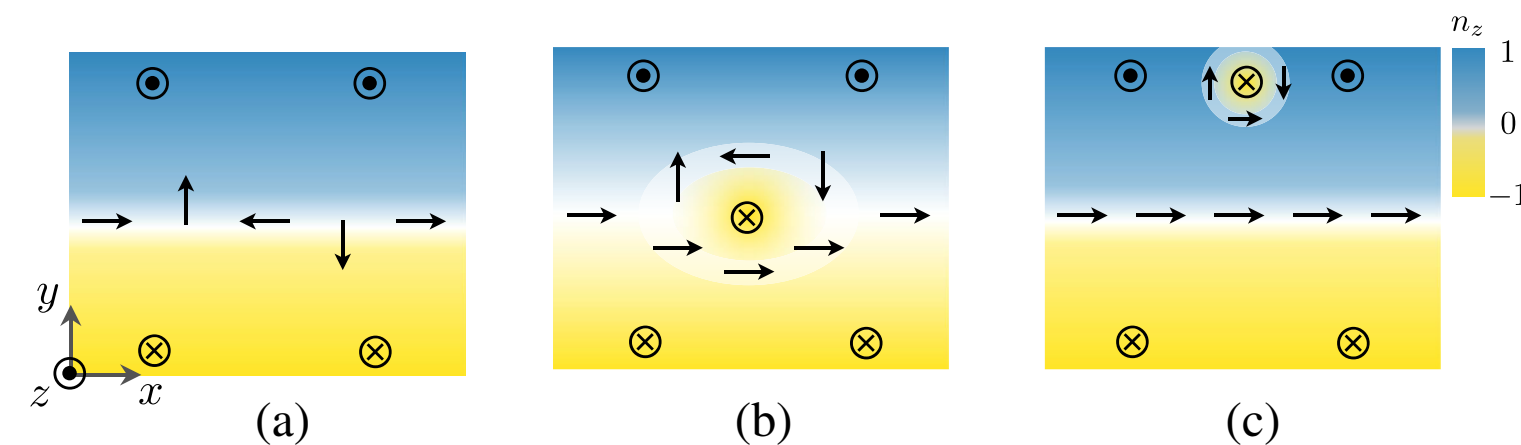


spin superfluid transport as a probe of quantum phases inaccessible to charge probes

PRL **119**, 047202 (2017) PHYSICAL REVIEW LETTERS week ending 28 JULY 2017

Magnetic Domain Walls as Hosts of Spin Superfluids and Generators of Skyrmions

Se Kwon Kim and Yaroslav Tserkovnyak
 Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

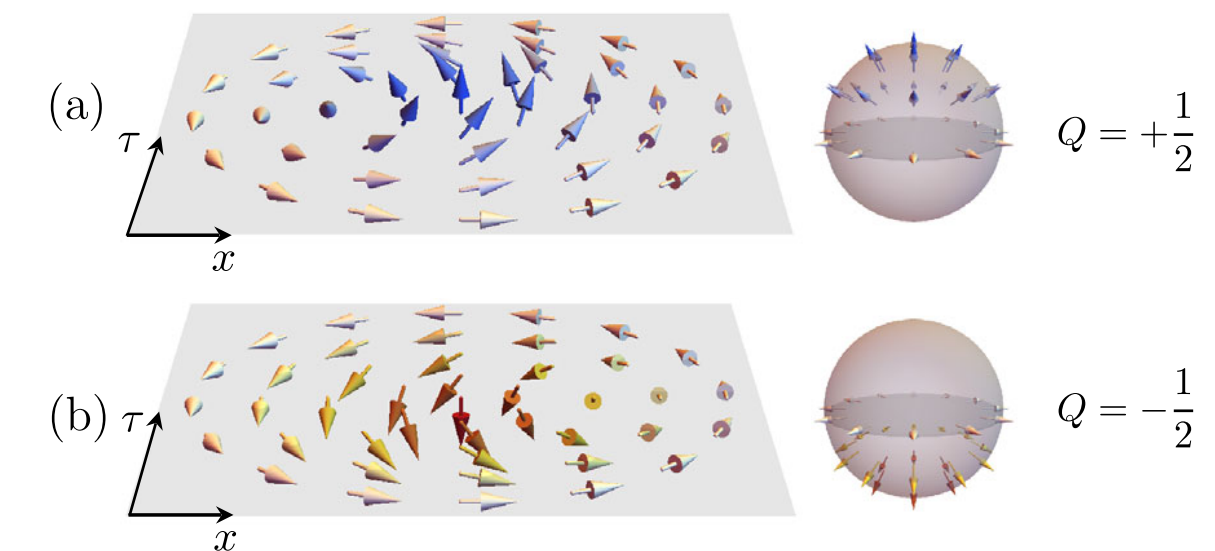


superfluidity along 1D conduits that are themselves topological defects

PRL **116**, 127201 (2016) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2016

Topological Effects on Quantum Phase Slips in Superfluid Spin Transport

Se Kwon Kim and Yaroslav Tserkovnyak
 Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA



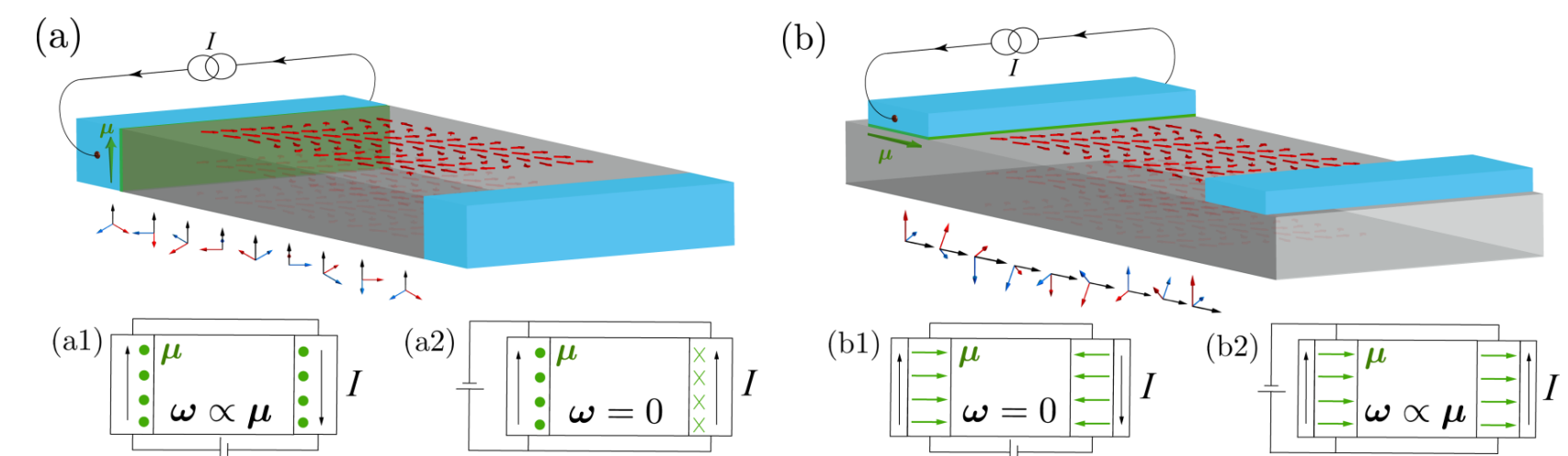
phase slip rates as a probe of collective thermal/quantum fluctuations

PHYSICAL REVIEW B **98**, 054424 (2018)

Editors' Suggestion

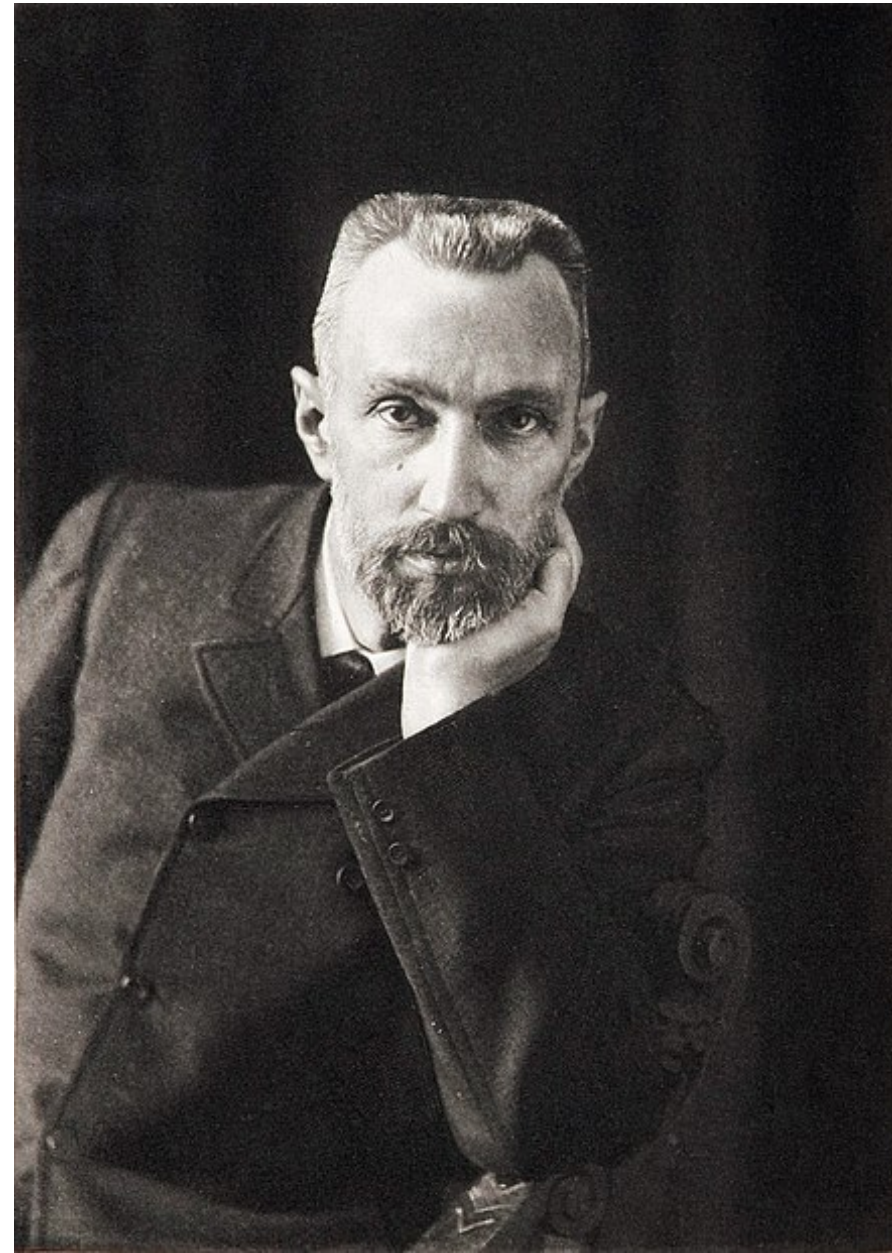
Spin hydrodynamics in amorphous magnets

Héctor Ochoa, Ricardo Zarzuela, and Yaroslav Tserkovnyak
 Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA



extending these ideas to richer noncollinear spin dynamics

General summary so far



Pierre Curie:

“Asymmetry creates the phenomenon”

energetics/symmetry-enabled injection and reciprocal detection



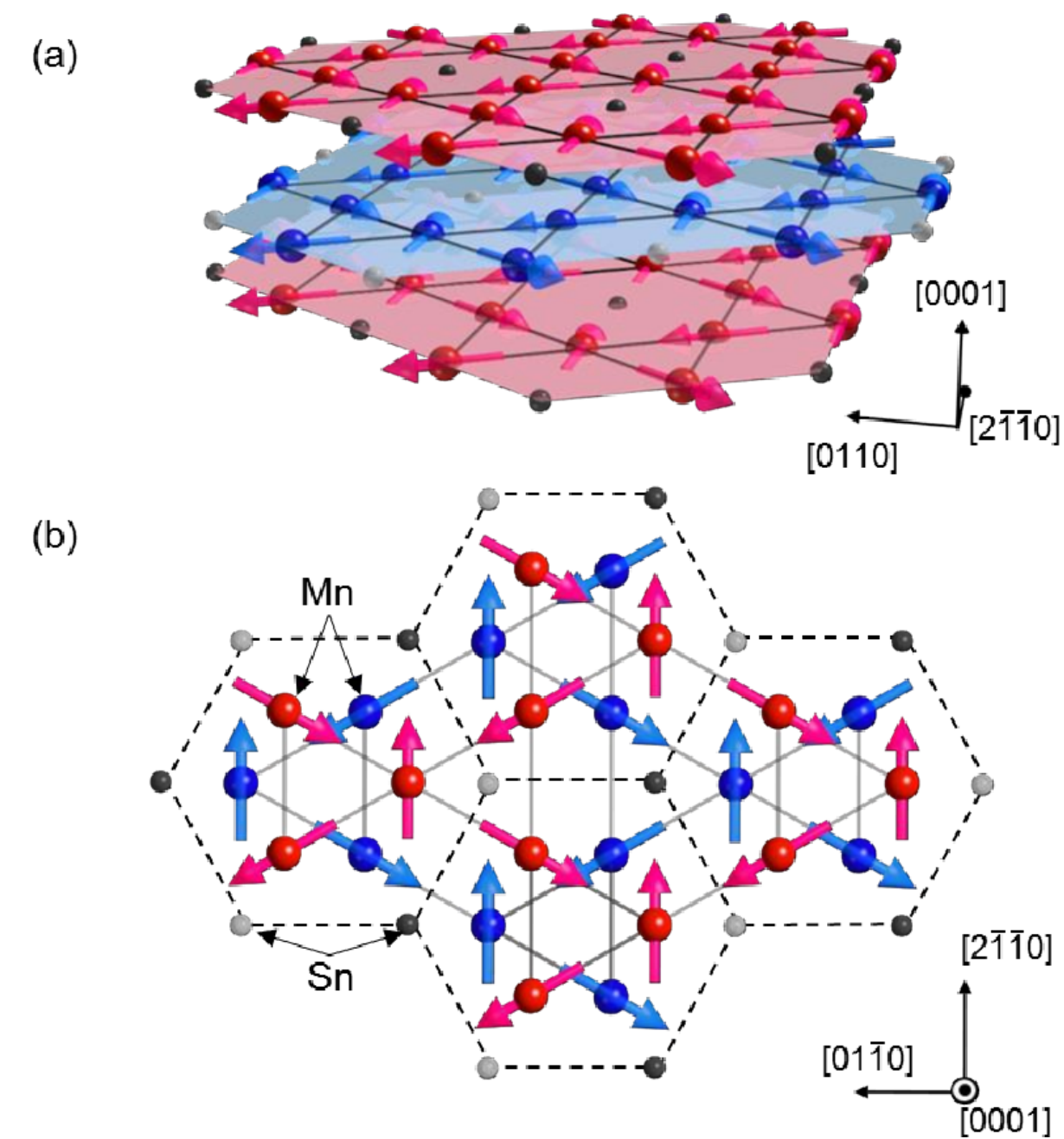
$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0$$

topological flavor

flux

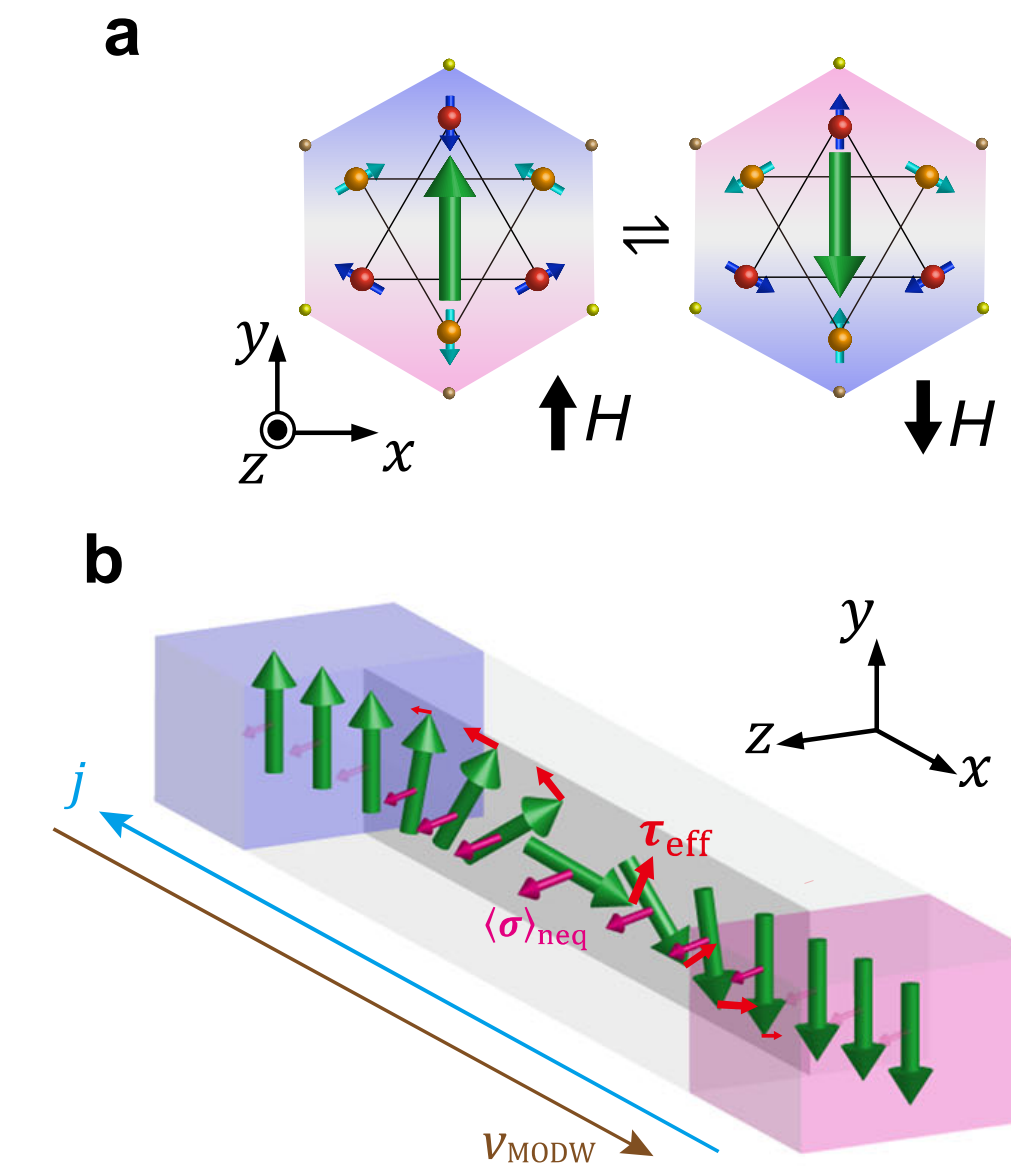
- Exploiting topological flavors of collective excitations in solid state to establish nonequilibrium transport phenomena
- Tailoring electrical means for their injection/detection
- Using this transport as a general-purpose probe of materials and a starting point for developing practical functionalities

A new promising materials platform for spin-winding dynamics: $\text{Mn}_3\text{Sn}(\text{Ge})$



Antiferromagnetic Weyl semimetal with planar (octupolar) spin texture free to rotate within the easy plane

Liu and Balents, *PRL* (2017)



Current-driven fast magnetic octupole domain-wall motion in noncollinear antiferromagnets

Wu, Otani, YT, et al., *Nat. Comm.* (2024)

Smooth driven octupole dynamics governed by energetics of winding:

The equations of motion are obtained from the free energy by recognizing canonical conjugacy of the in-plane angle θ (a generalized coordinate) and the net out-of-plane spin density ρ (the associated conjugate momentum - being the generator of the xy planar spin rotations):

$$\left[\partial_t \theta = \frac{\partial F}{\partial \rho} = \frac{\rho}{\chi} \qquad \partial_t \rho = -\frac{\partial F}{\partial \theta} - \frac{\partial R}{\partial \dot{\theta}} + \tau = A \nabla^2 \theta - n \lambda \sin(n\theta) - \alpha s \partial_t \theta + \tau \right]$$

current-induced spin-transfer torque: $\tau = -\beta s \mathbf{v} \cdot \nabla \theta$ where $\mathbf{v} = \mathbf{j}/en_e$

results in net (dissipation plus work) power $\partial_t F = \partial_t \theta \partial_\theta F + \partial_t \rho \partial_\rho F = -\alpha s \partial_t \theta D_t \theta$

in terms of advective derivative $\boxed{D_t \equiv \partial_t + \eta \mathbf{v} \cdot \nabla}$ where $\eta \equiv \beta/\alpha$

looking for rigid translations $\theta \rightarrow \theta(\mathbf{r} - \mathbf{u}t)$ and setting steady-state power $P \rightarrow 0$

$$P = -\alpha s \int d^3r (\mathbf{u} \cdot \nabla \theta) [(\mathbf{u} - \eta \mathbf{v}) \cdot \nabla \theta]$$

we finally get for the domain-wall velocity $\boxed{\mathbf{u} = \eta \mathbf{v}}$

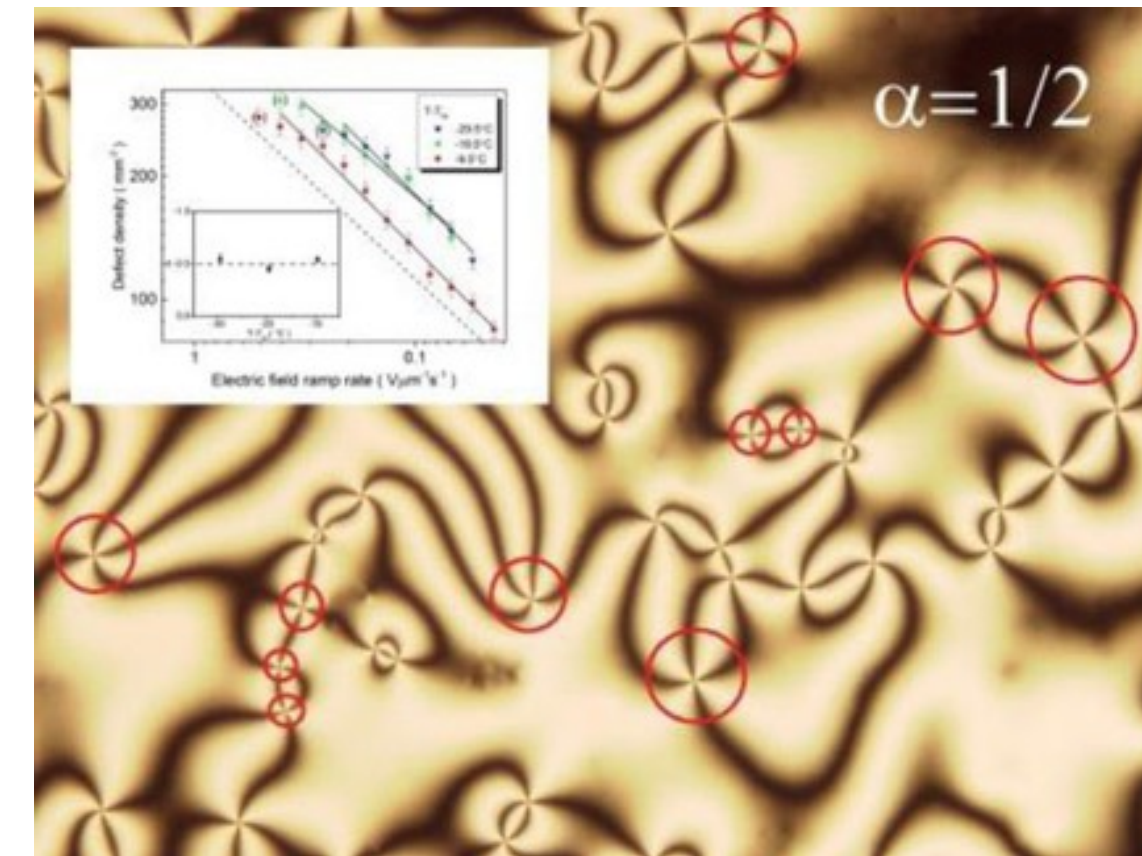
Next steps: generalizations and broader themes

- **Going beyond superfluid-like planar winding**
 - *Dimensionality-dependent generalizations: vortices, baby (2D) skyrmions, hedgehogs, Shankar (3D) skyrmions, hopfions etc.*
- **Classifications based on topology vs differential geometry**
 - *Conservation laws based on topological invariance (absolute vs relative homotopies) vs generalized Stokes' theorems*
- **Curved geometries as a structural handle on symmetries**
 - *Do the conservation laws survive nontrivial shapes? Can we exploit it?*
- **What about thermal and quantum fluctuations?**
 - *Local vs nonlocal topological flavors, quantum spin-lattice constructions and field theory*
- **Non-Abelian topological flavors**
 - *Can we nucleate and transport topological charges that cannot be counted?*

Nature is replete with rich topological structures



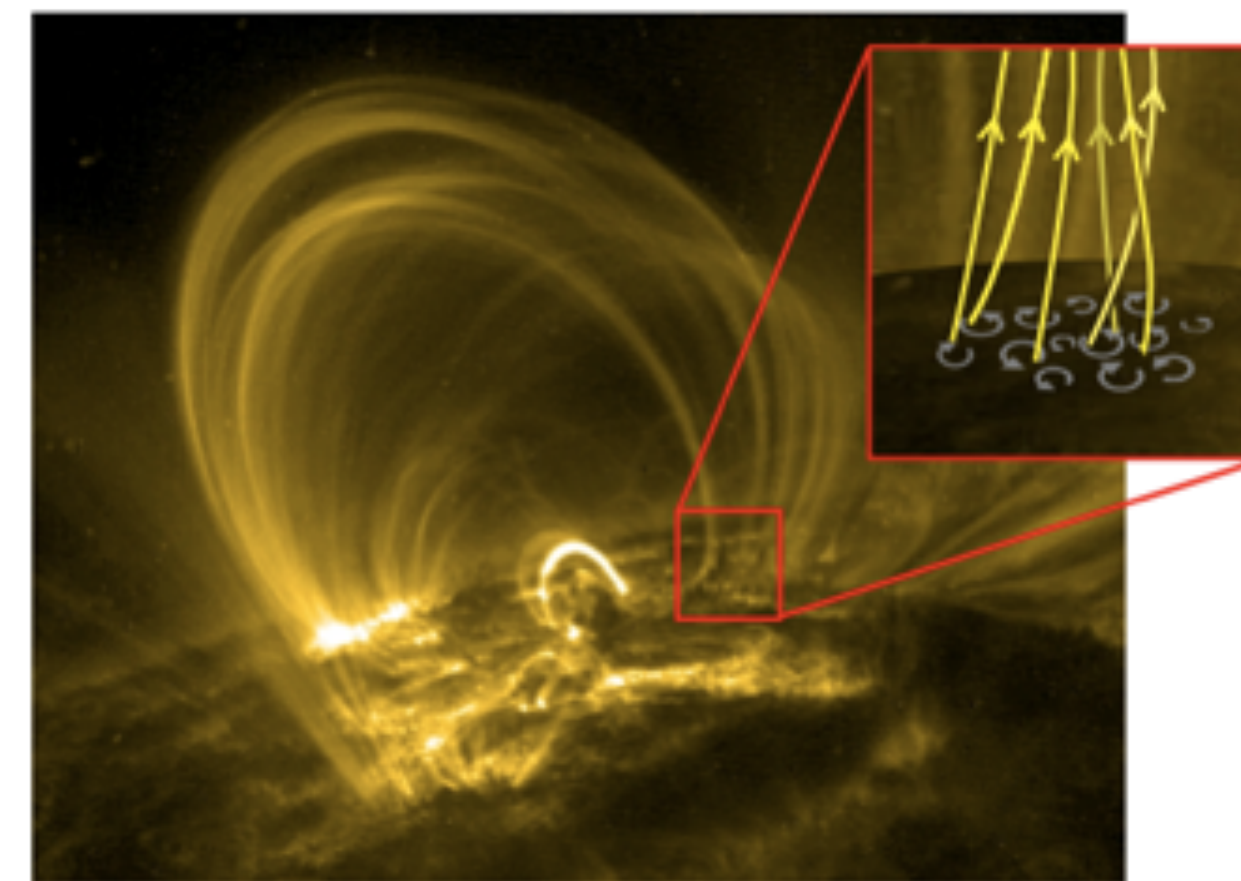
Crystalline defects



Kibble-Zurek mechanism in nematics



Rib vortices in ocean waves



Magnetic flux ropes in sun corona

Unifying themes for their studies and utilization?

Electrical injection/detection of vorticity provides an illuminating example

Energetics (thermodynamics) and symmetries:

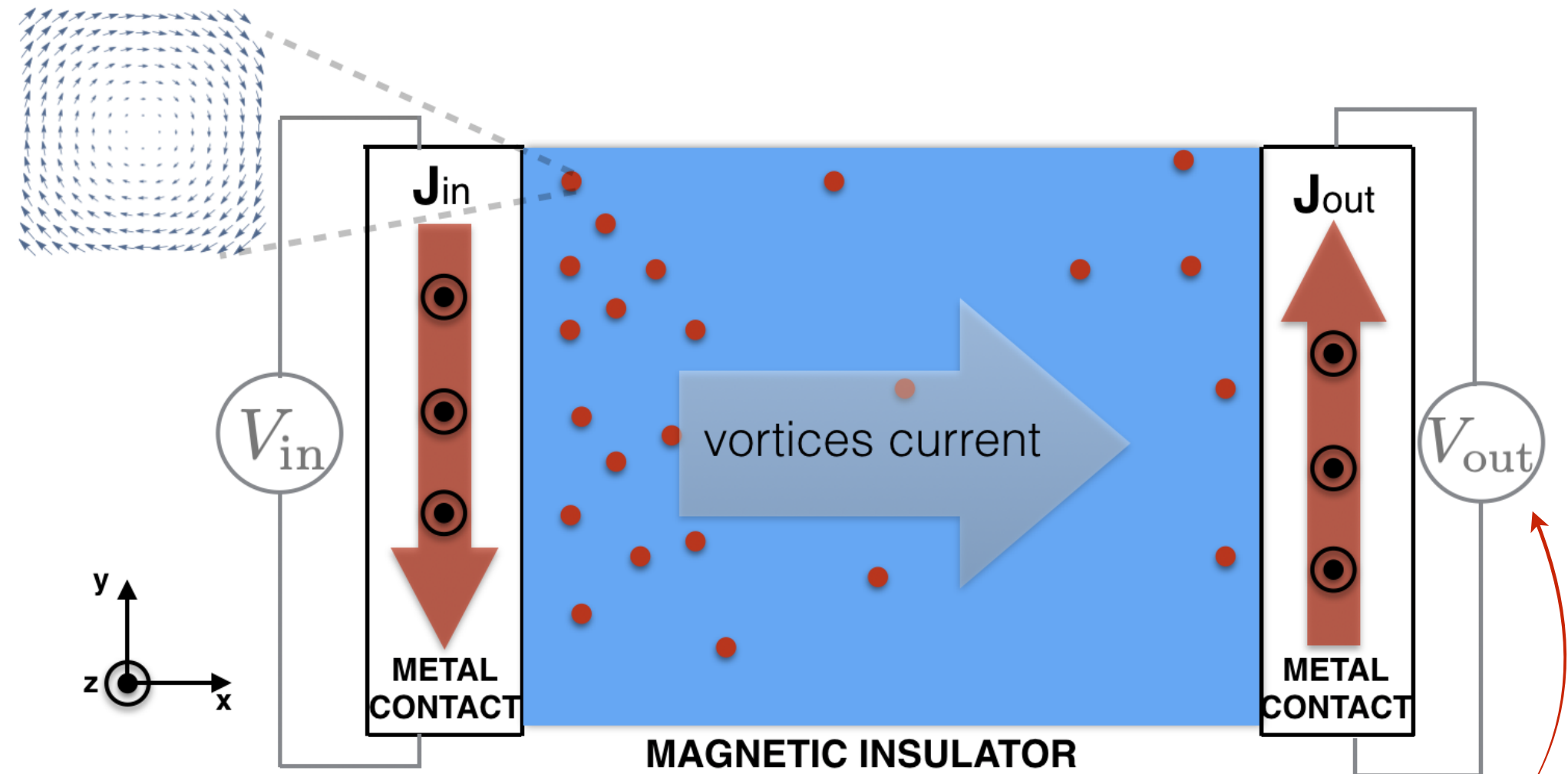
spin torque by the input current at the left interface:

$$\boldsymbol{\tau} = \eta \mathbf{M} \cdot \mathbf{m} (\mathbf{J} \cdot \nabla) \mathbf{m}$$

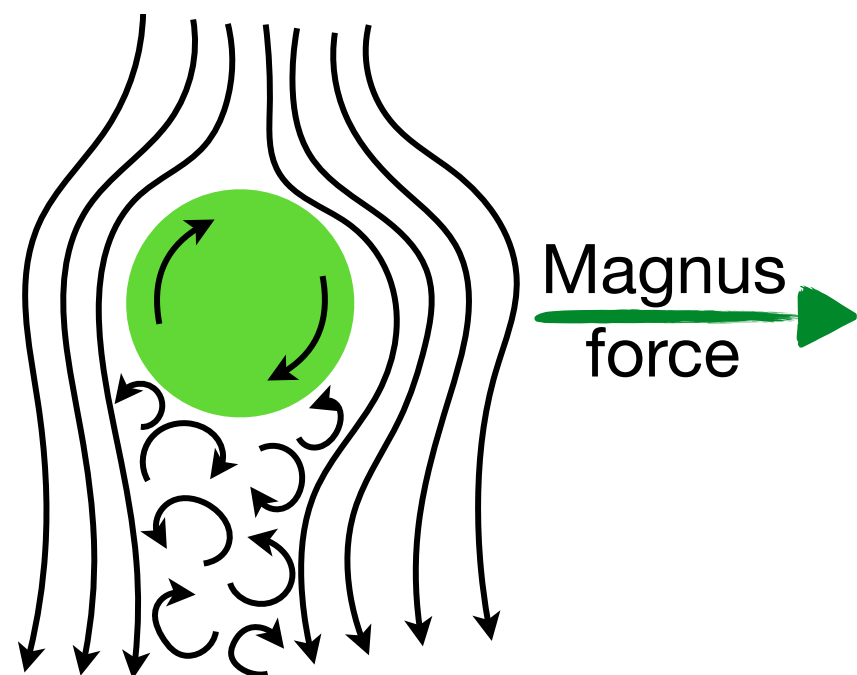
$\mathbf{M} \propto \mathbf{z}$ - magnetization of the metal contact

$\mathbf{m}(x, y, t)$ - magnetic texture of the insulator

Zou, Kim, and YT, *PRB* (2019)



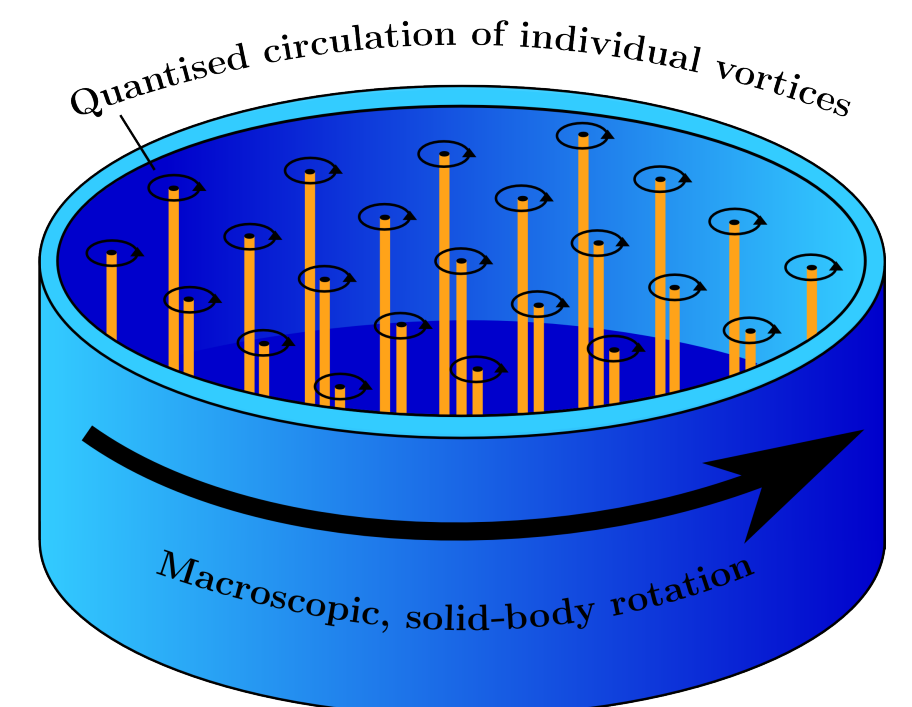
Onsager-reciprocal
motive force



$$\dot{W} = \int dy \boldsymbol{\tau} \cdot (\mathbf{m} \times \partial_t \mathbf{m}) \rightarrow \eta \mathbf{z} \cdot \mathbf{J} \times \mathbf{j}$$

charge current

vortex flux



Winding-vorticity interconversion

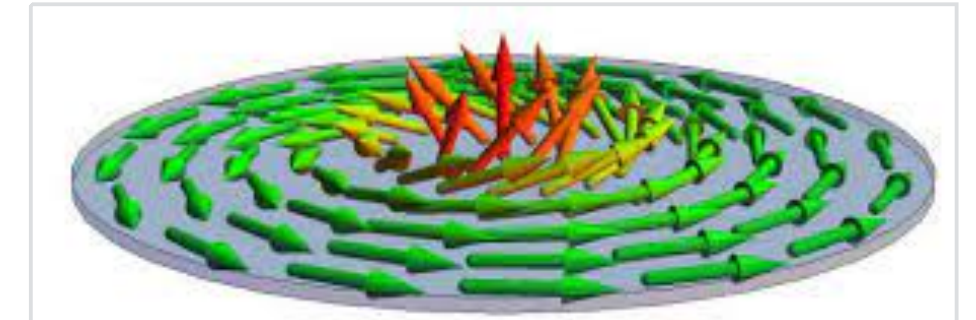
In terms of an *arbitrary* three-component vector field $\mathbf{m}(\mathbf{r}, t)$:

2D vorticity:

$$\left. \begin{aligned} \rho_v &= \frac{\mathbf{z} \cdot \partial_x \mathbf{m} \times \partial_y \mathbf{m}}{\pi} \\ j_v^i &= \frac{\epsilon^{ij} \mathbf{z} \cdot \partial_j \mathbf{m} \times \partial_t \mathbf{m}}{\pi} \end{aligned} \right\} \begin{aligned} &\partial_t \rho_v + \nabla \cdot \mathbf{j}_v = 0 \\ &\text{vorticity conservation} \end{aligned}$$

1D winding:

$$\left. \begin{aligned} \rho_w &= \frac{\mathbf{z} \cdot \mathbf{m} \times \partial_x \mathbf{m}}{2\pi} \\ j_w &= -\frac{\mathbf{z} \cdot \mathbf{m} \times \partial_t \mathbf{m}}{2\pi} \end{aligned} \right\} \partial_t \int_{-}^{+} dx \rho_w + (j_w^{+} - j_w^{-}) = \int_{-}^{+} dx j_v^y$$

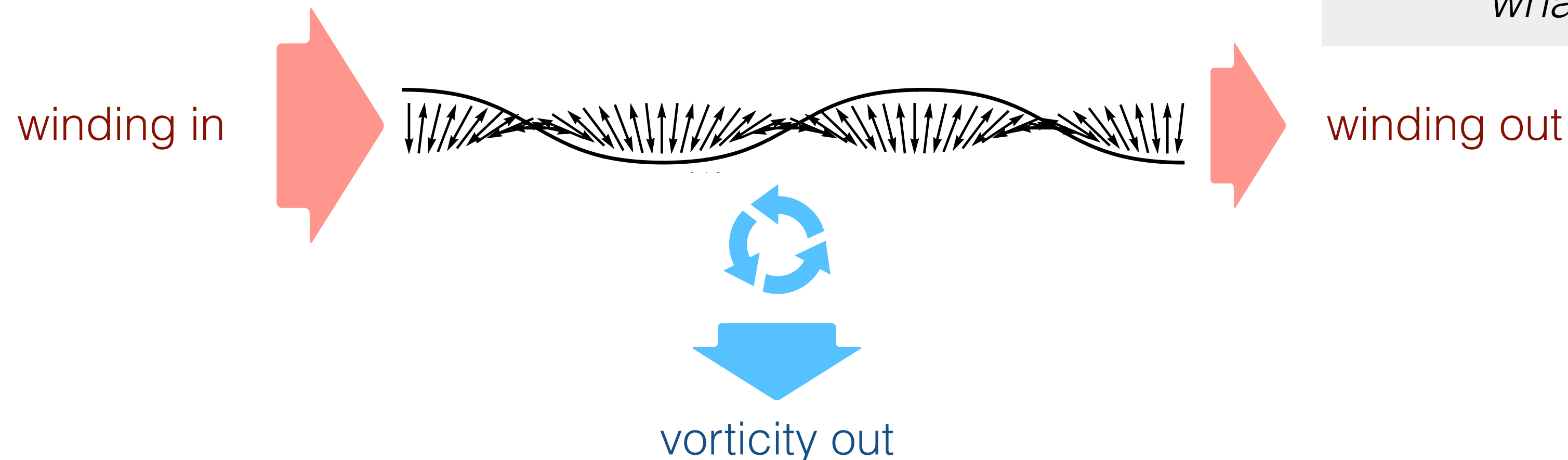


$$Q_v = \int dx dy \rho_v \rightarrow 1$$

this charge is strictly quantized only when $|\mathbf{m}| \equiv 1$ and $m_z = 0$ on the boundary

the *vorticity conservation* law is valid either way!

what about topology?



Underlying homotopic structure captures the essence of winding/vorticity

...in the strongly-ordered phase

The topological theory of defects in ordered media*†

N. D. Mermin

Laboratory of Atomic and Solid State Physics Cornell University, Ithaca, New York 14853

Aspects of the theory of homotopy groups are described in a mathematical style closer to that of condensed matter physics than that of topology. The aim is to make more readily accessible to physicists the recent applications of homotopy theory to the study of defects in ordered media. Although many physical examples are woven into the development of the subject, the focus is on mathematical pedagogy rather than on a systematic review of applications.

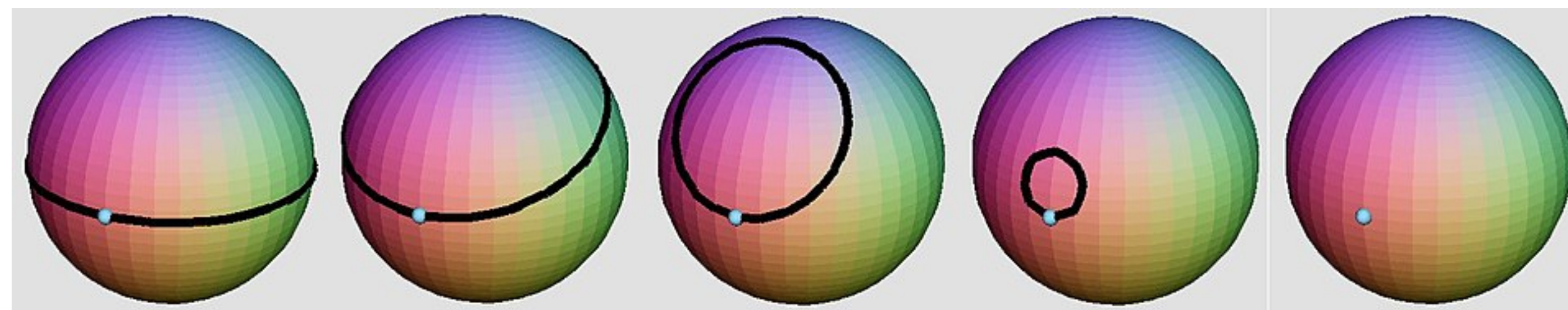
Reviews of Modern Physics, Vol. 51, No. 3, July 1979

fundamental (homotopy) group $\pi_1(M)$ captures topologically distinct smooth textures in 1D or singular point defects in 2D

2nd-order homotopy group $\pi_2(M)$ captures topologically distinct smooth textures in 2D or singular point defects in 3D



Henri Poincaré



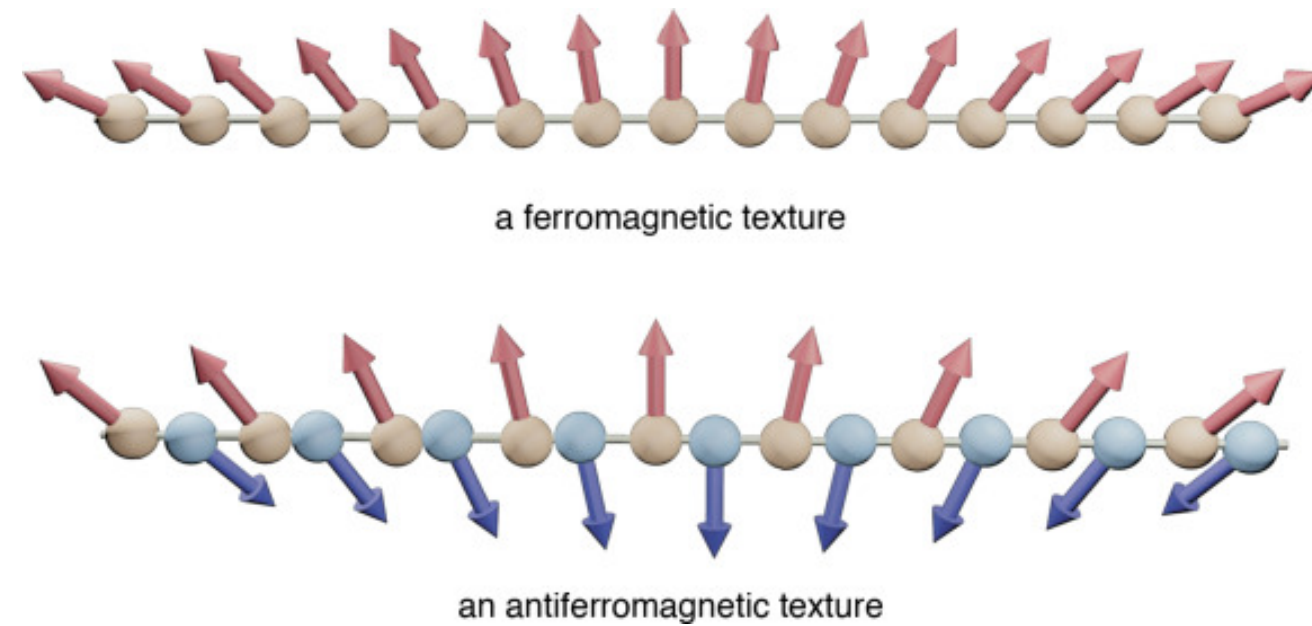
$$\pi_1(S^2) = 0$$

no superflows (topologically stable textures) in 1D
Heisenberg magnets

The elementary nontrivial cases are

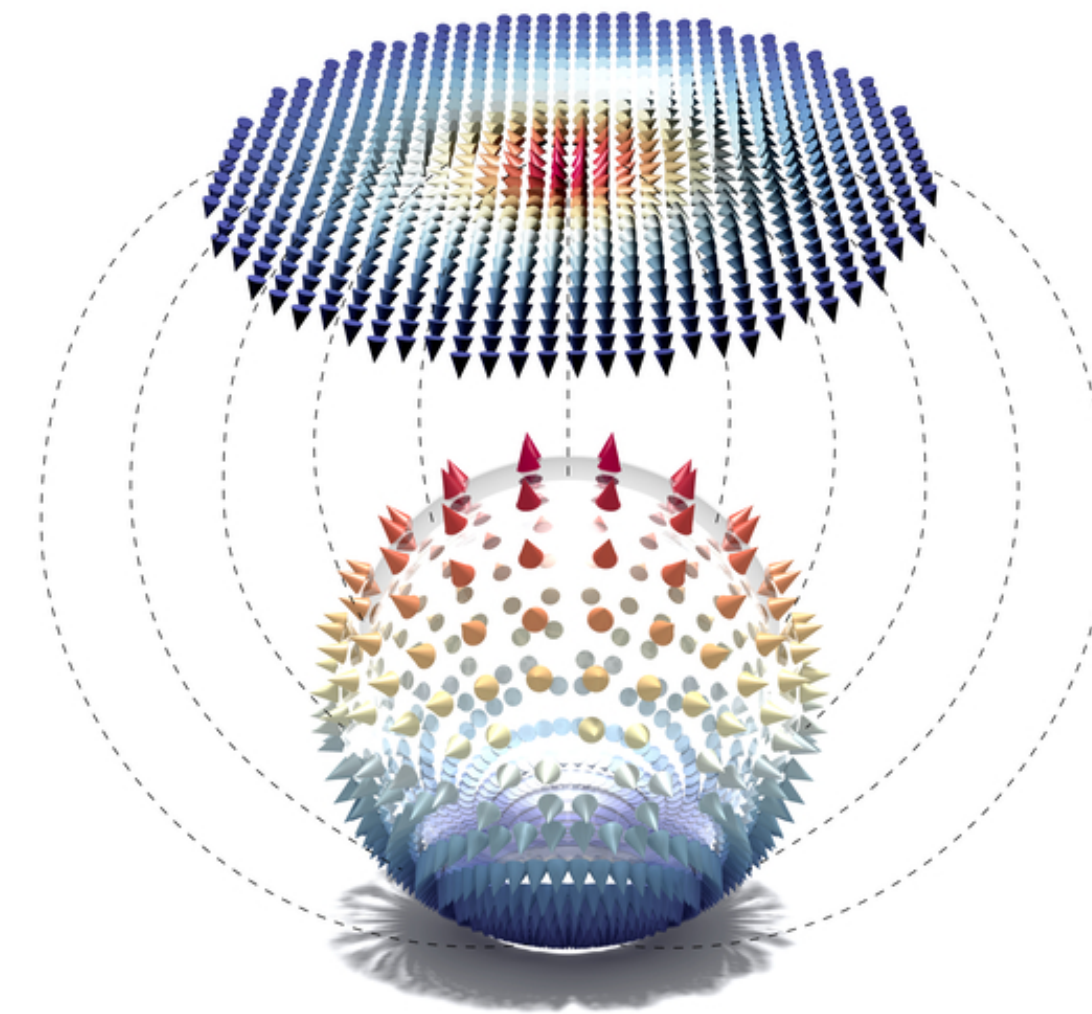
$$\pi_1(S^1) = \mathbb{Z}$$

easy-plane superflows in
1D & vortices in 2D
(superconductors or easy-
plane magnets)



$$\pi_2(S^2) = \mathbb{Z}$$

skyrmions in 2D &
hedgehogs in 3D
(Heisenberg magnets)



Cortes, Beg & Fangohr (2018)

fundamental (homotopy) group $\pi_1(M)$ captures topologically distinct smooth textures in 1D or singular point defects in 2D

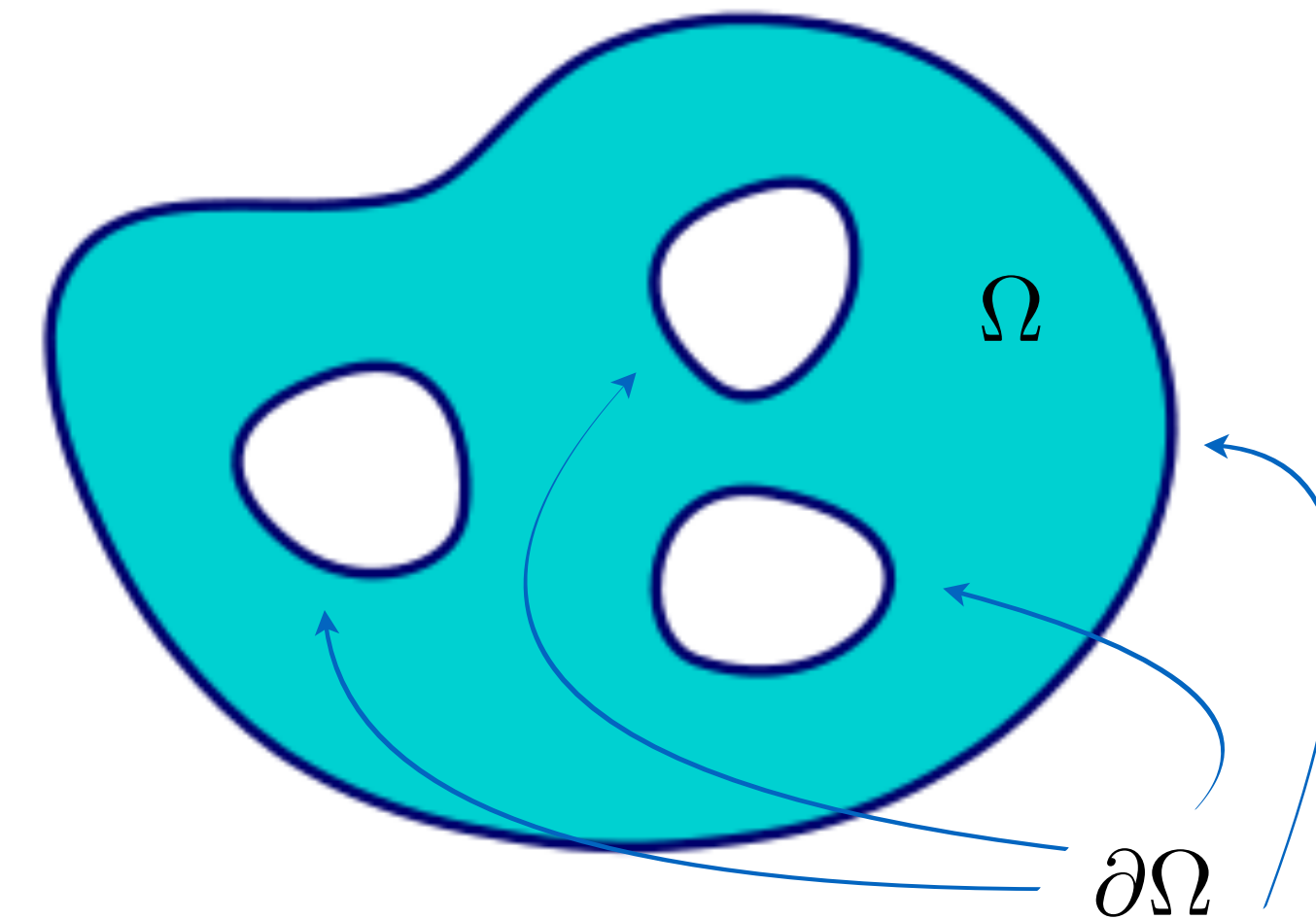
2nd-order homotopy group $\pi_2(M)$ captures topologically distinct smooth textures in 2D or singular point defects in 3D

Topology or Stokes' theorem?

The vorticity of a strongly fluctuating vector field $\mathbf{m}(\mathbf{r}, t)$ is governed by the Stokes' theorem:

$$\oint_{\partial\Omega} \omega = \int_{\Omega} d\omega$$

p -form ω
 oriented boundary $\partial\Omega$
 oriented region in a manifold Ω
 differential-geometric Stokes' theorem

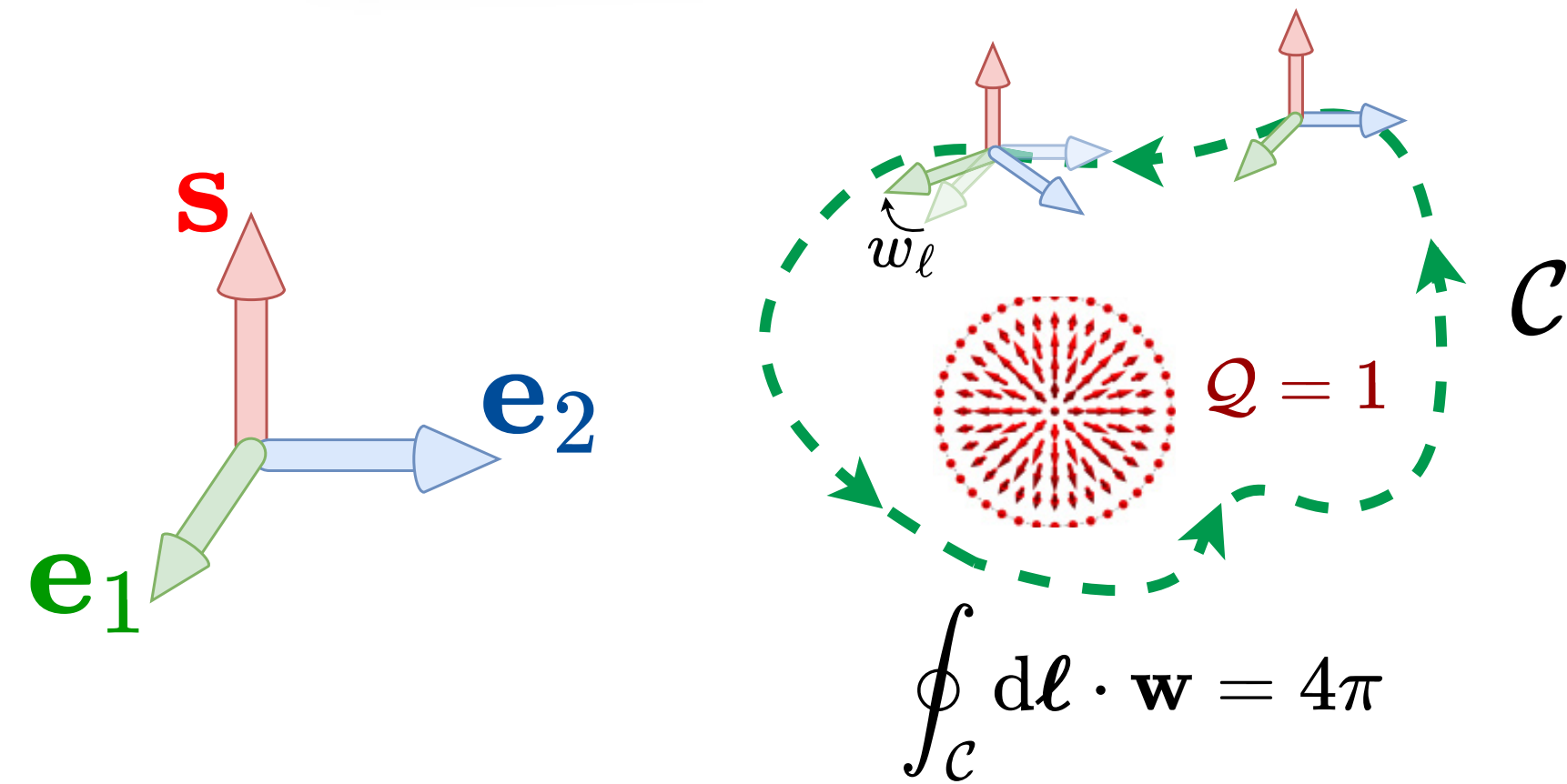


$$p = 1 : \quad \oint d\mathbf{l} \cdot \boldsymbol{\rho}_w = \int dA \rho_v \quad \text{where} \quad \boldsymbol{\rho}_w \cdot \mathbf{x} = \frac{\mathbf{z} \cdot \mathbf{m} \times \partial_x \mathbf{m}}{2\pi} \quad \text{and} \quad \rho_v = \frac{\mathbf{z} \cdot \partial_x \mathbf{m} \times \partial_y \mathbf{m}}{\pi}$$

Note: Stokes theorem always insures a (differential-geometric) bulk-boundary correspondence and the associated conservation law

(We will benefit from this more formal construction in curved spaces)

$SO(3)$ vortices of a triplet superconductor provide a peculiar example

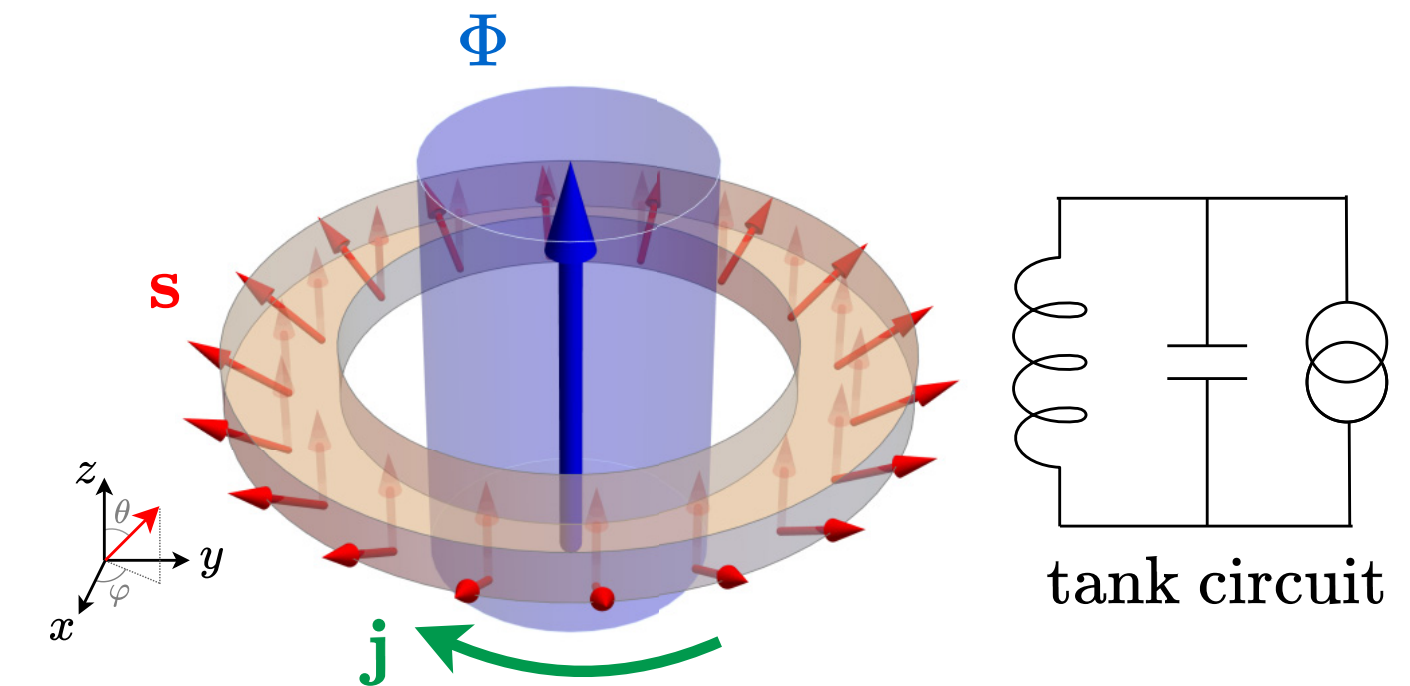


$$\mathbf{s} = \mathbf{e}_1 \times \mathbf{e}_2, \quad \mathbf{w} = (\nabla \mathbf{e}_2) \cdot \mathbf{e}_1$$

Net winding can change smoothly in 4π increments

$$\mathcal{W} = \int_{\partial \mathcal{A}} d\ell \cdot \mathbf{w} = \int_{\mathcal{A}} dA \varrho_{\text{sk}}(\mathbf{s})$$

Mermin-Ho relation (follows from Stokes' theorem)



spin-triplet SQUID

(nonsingular phase slips: coreless vortices)

$$\mathcal{F}[\mathbf{s}, w] = \int d\ell \left[\frac{A}{2} (\partial_\ell \mathbf{s})^2 + \frac{1}{2\chi_w} \left(w - \frac{qA_\ell}{\hbar c} \right)^2 \right]$$

free-energy is constructed in terms of physical and topological densities

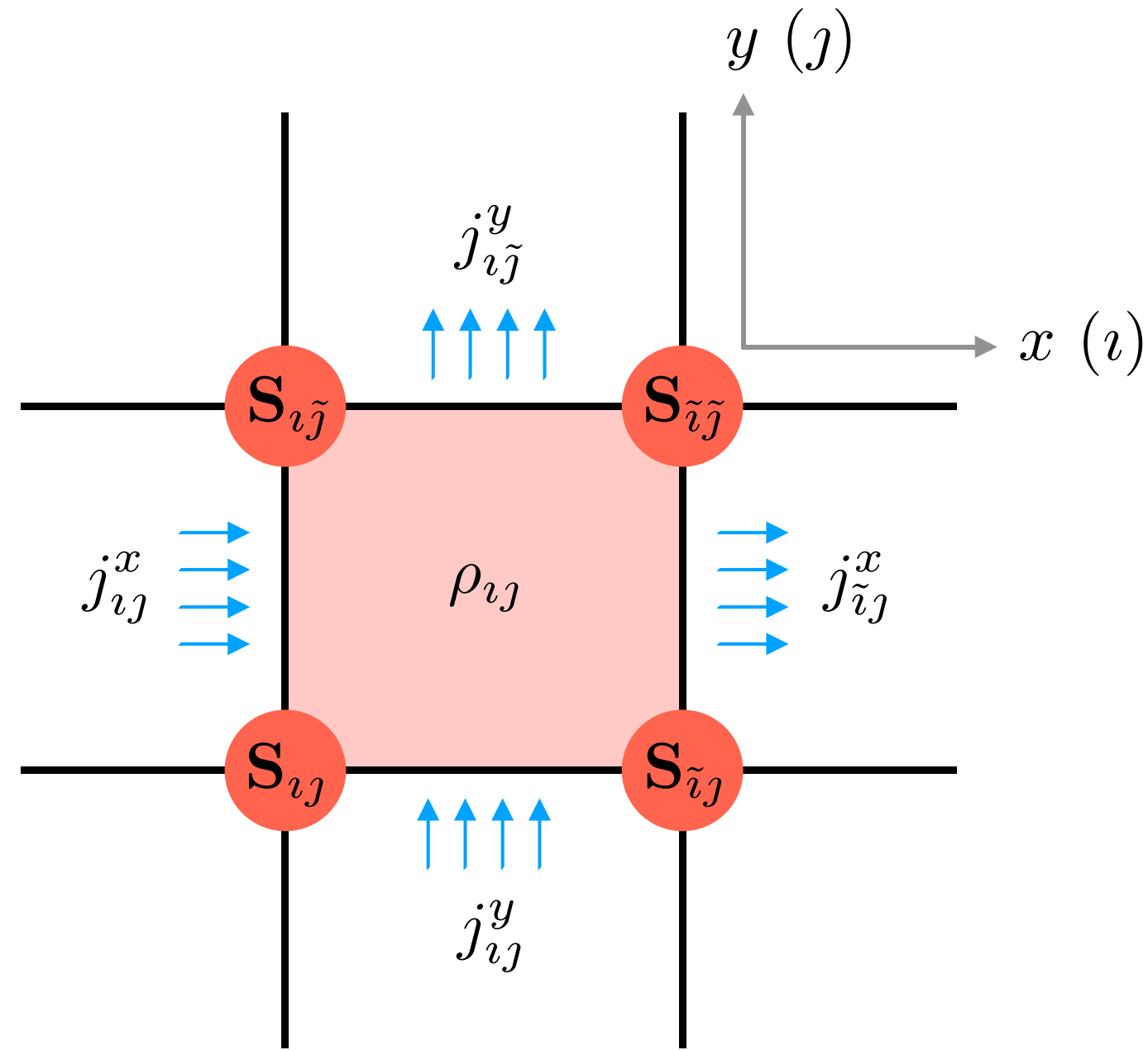
Absolute homotopy misses the associated conserved quantity since

$$\pi_1[SO(3)] = \pi_2[SO(3)] = 0$$

The topological fingerprints are contained here in the *relative* homotopy:

$$\pi_2[SO(3), U(1)] = \mathbb{Z}$$

The vorticity is robust against quantum fluctuations and lattice effects



$$\partial_t \rho_{ij} + \frac{j_{\tilde{i}j}^x - j_{ij}^x + j_{i\tilde{j}}^y - j_{ij}^y}{a} = 0$$

- Vorticity per plaquette is given by the (z component of the) **vector chirality**: quantum version of the winding around the plaquette

$$\rho_{ij} = \frac{\mathbf{z} \cdot \mathbf{c}_{ij}}{2\pi a^2} \quad \text{where} \quad \mathbf{c}_{ij} \equiv \frac{1}{S^2} \sum_l \mathbf{S}_l \times \mathbf{S}_{\tilde{l}}$$

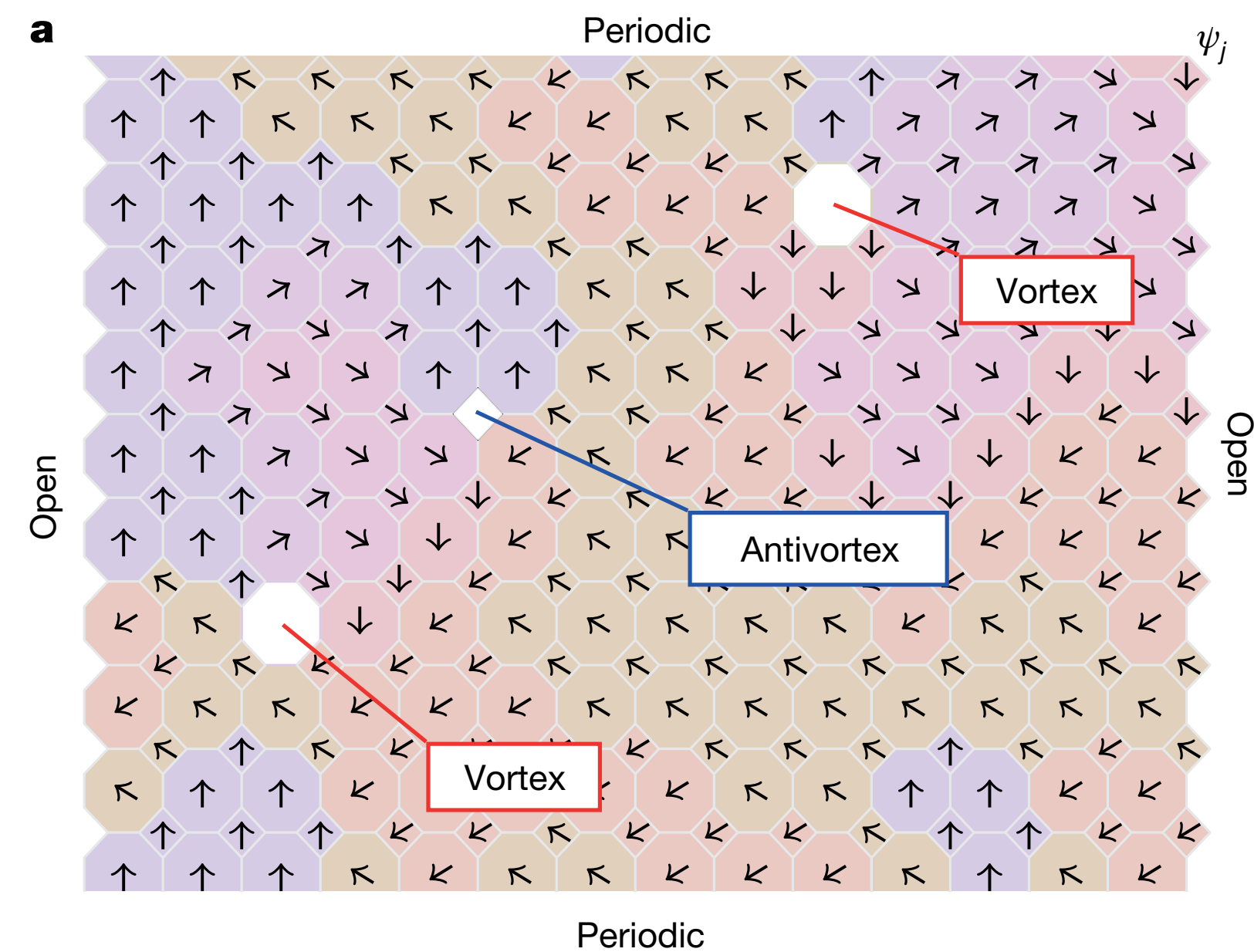
- For smooth classical textures, this reproduces the previous continuum version

Quantum annealers offer another experimental platform

Observation of topological phenomena in a programmable lattice of 1,800 qubits

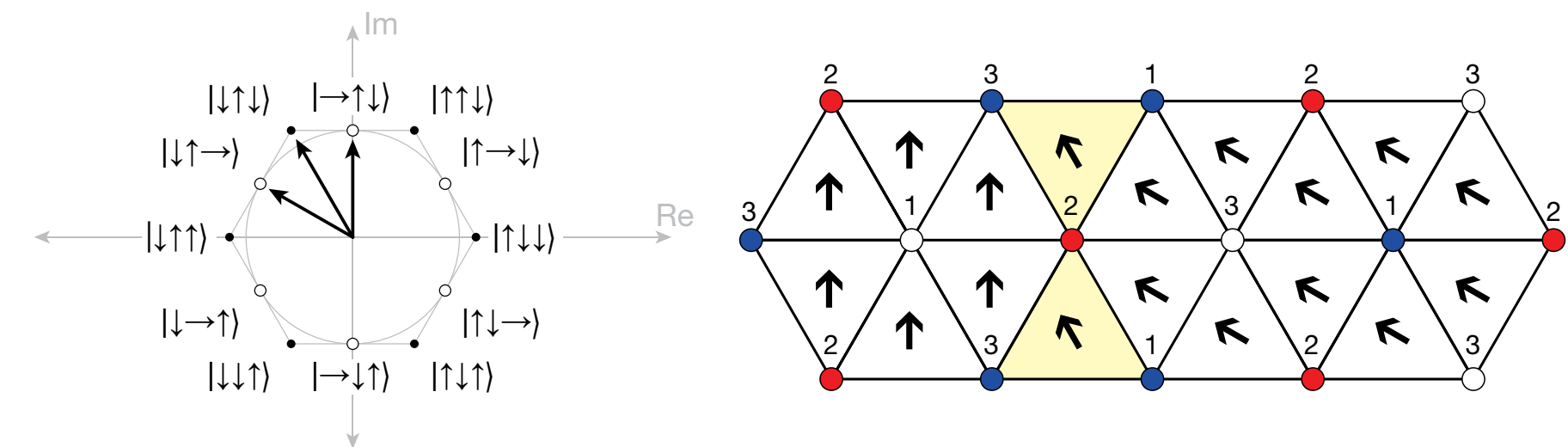
Andrew D. King^{1*}, Juan Carrasquilla², Jack Raymond¹, Isil Ozfidan¹, Evgeny Andriyash¹, Andrew Berkley¹, Mauricio Reis¹, Trevor Lanting¹, Richard Harris¹, Fabio Altomare¹, Kelly Boothby¹, Paul I. Bunyk¹, Colin Enderud¹, Alexandre Fréchet¹, Emile Hoskinson¹, Nicolas Ladizinsky¹, Travis Oh¹, Gabriel Poulin-Lamarre¹, Christopher Rich¹, Yuki Sato¹, Anatoly Yu. Smirnov¹, Loren J. Swenson¹, Mark H. Volkmann¹, Jed Whittaker¹, Jason Yao¹, Eric Ladizinsky¹, Mark W. Johnson¹, Jeremy Hilton¹ & Mohammad H. Amin^{1,3}

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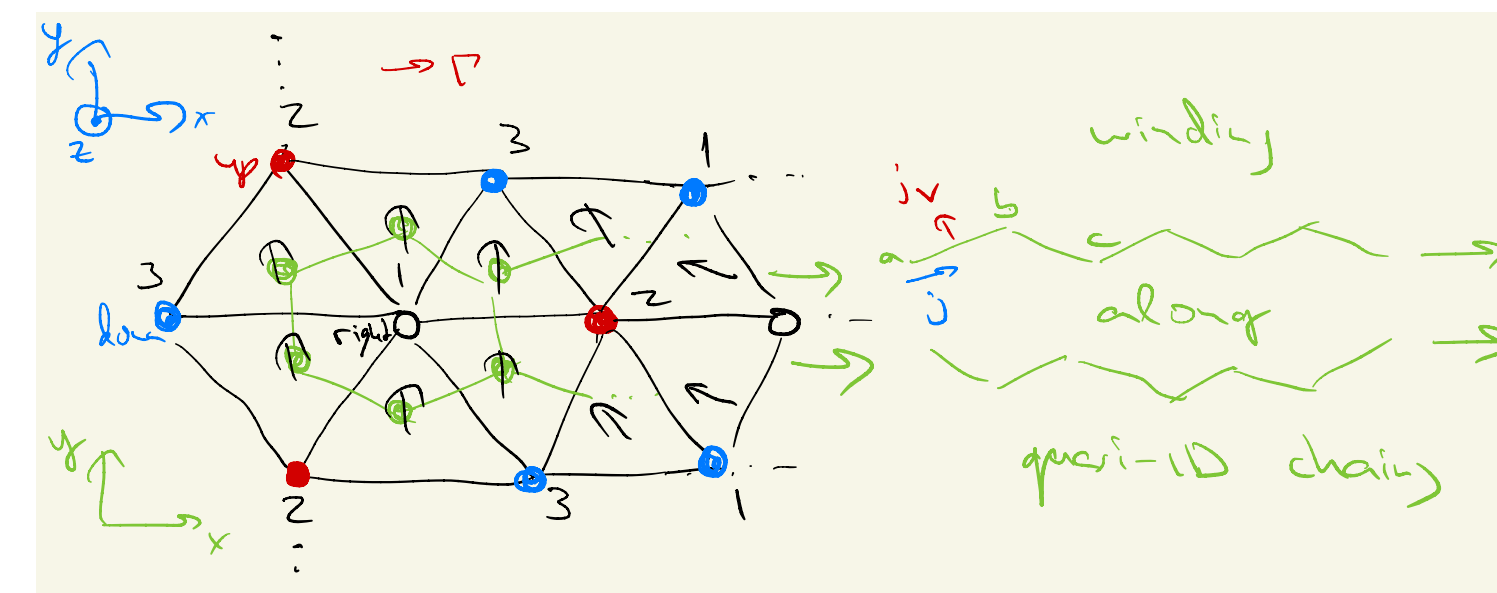


$$T < T_{KT}$$

vortices bind with antivortices:
vorticity insulator



$$\psi = \hat{\sigma}_1^z + \hat{\sigma}_2^z e^{2\pi i/3} + \hat{\sigma}_3^z e^{-2\pi i/3}$$

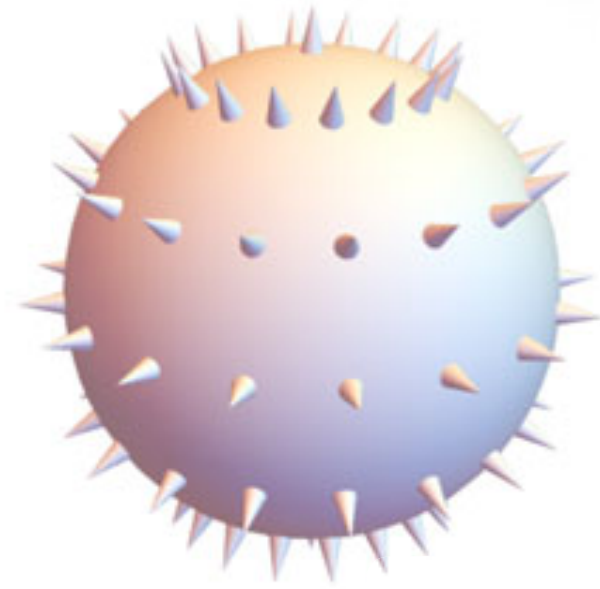


$$T > T_{KT}$$

(anti)vortices unbind and proliferate:
vorticity metal (two-component plasma)

3D version of vortices: Hedgehogs

- The continuum hedgehog hydrodynamics

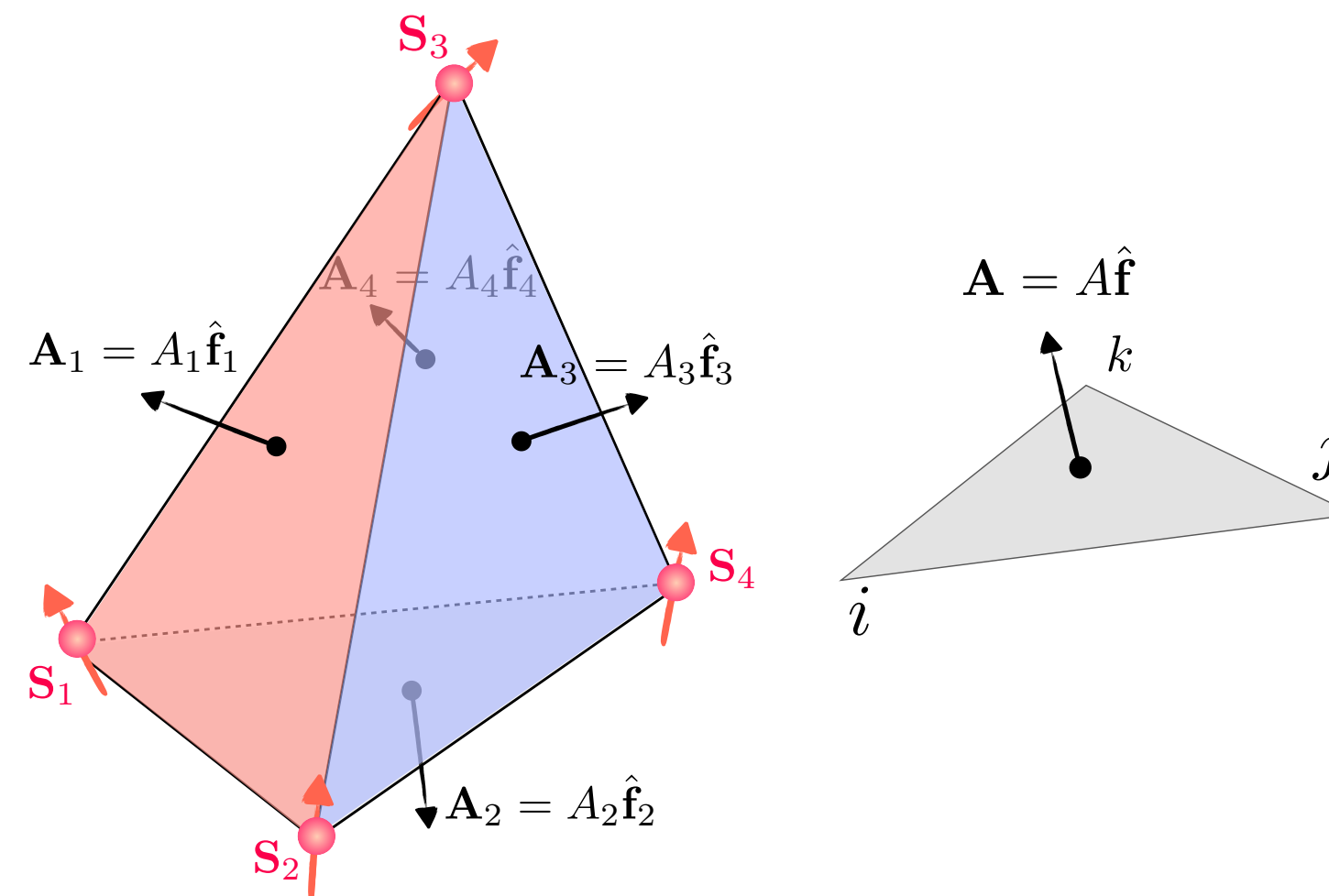


$$\mathbf{m}(\mathbf{r}, t) : \quad \mathbf{m}, \mathbf{r} \in \mathbb{R}^3$$

$$j^\mu = \epsilon^{\mu\alpha\beta\gamma} \partial_\alpha \mathbf{m} \cdot (\partial_\beta \mathbf{m} \times \partial_\gamma \mathbf{m}) / 8\pi, \quad \partial_\mu j^\mu = 0$$

the conserved (generally nonquantized) charge corresponds to the skyrmion density on the enclosing surface, according to the higher-dimensional Stokes' theorem

has a natural quantum underpinning on an arbitrary lattice, in terms of **scalar spin chirality**:

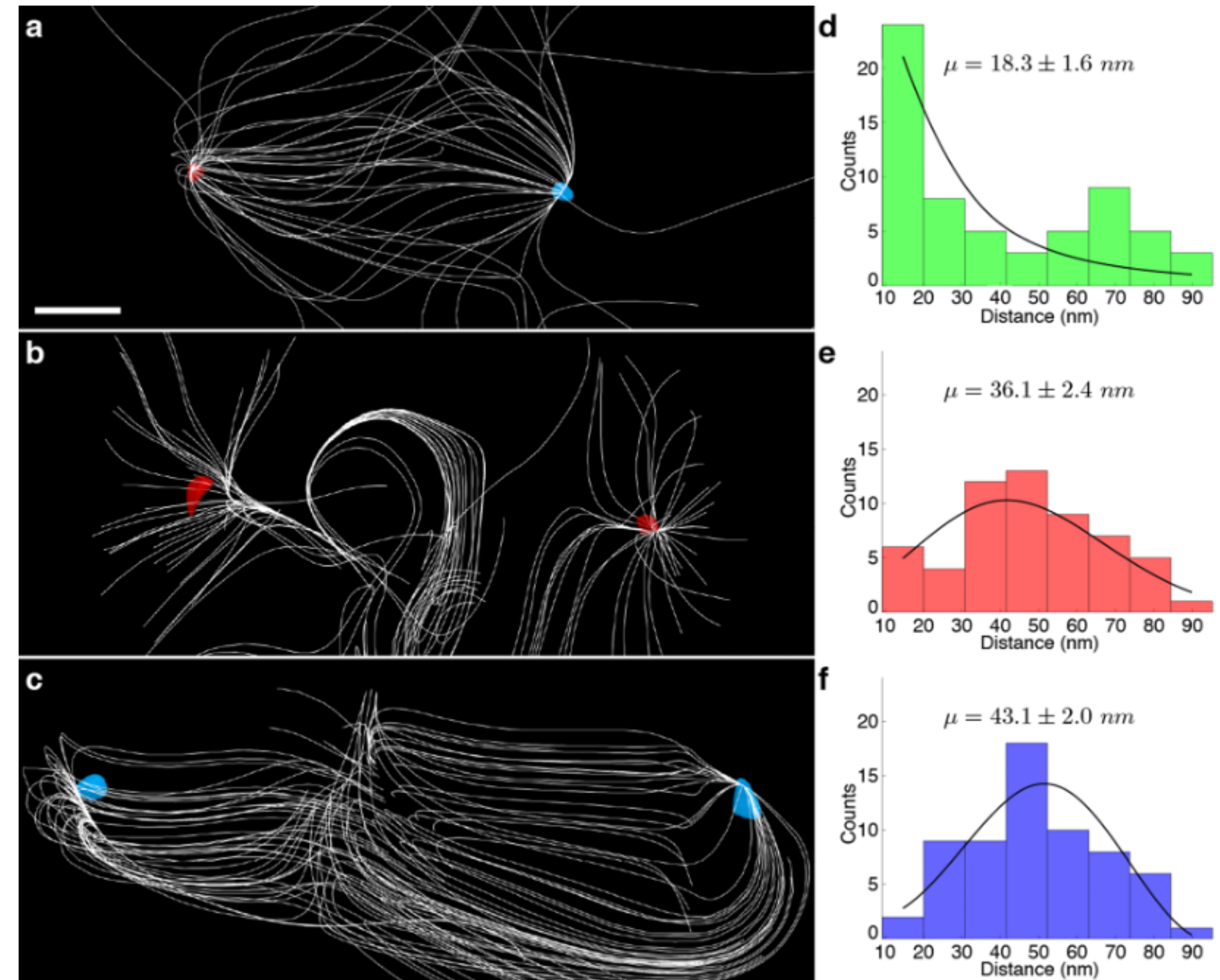
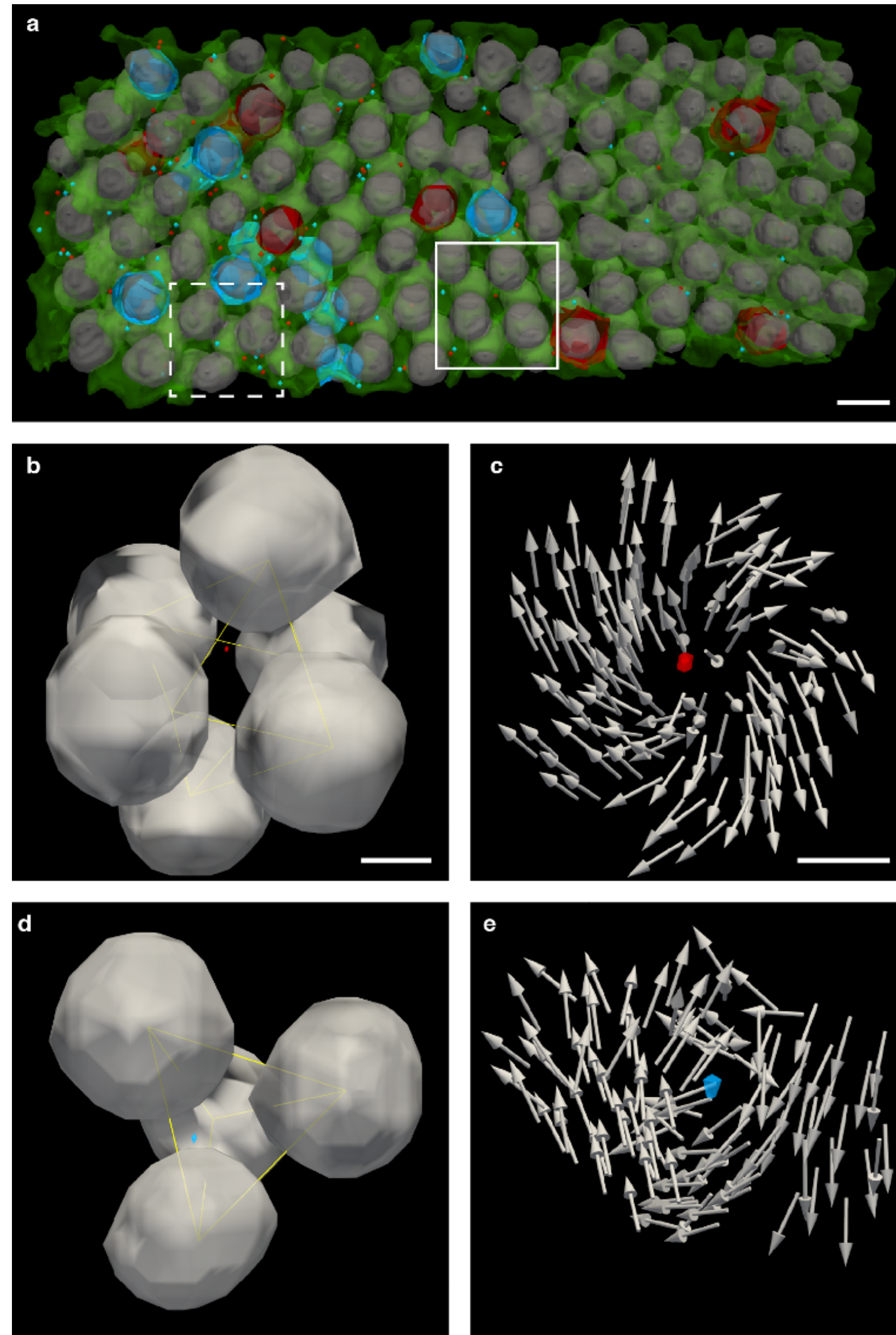


$$\rho = \frac{c_{123} + c_{142} + c_{243} + c_{134}}{8\pi V}$$

where

$$c_{ijk} = \frac{\mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k}{S^3}$$

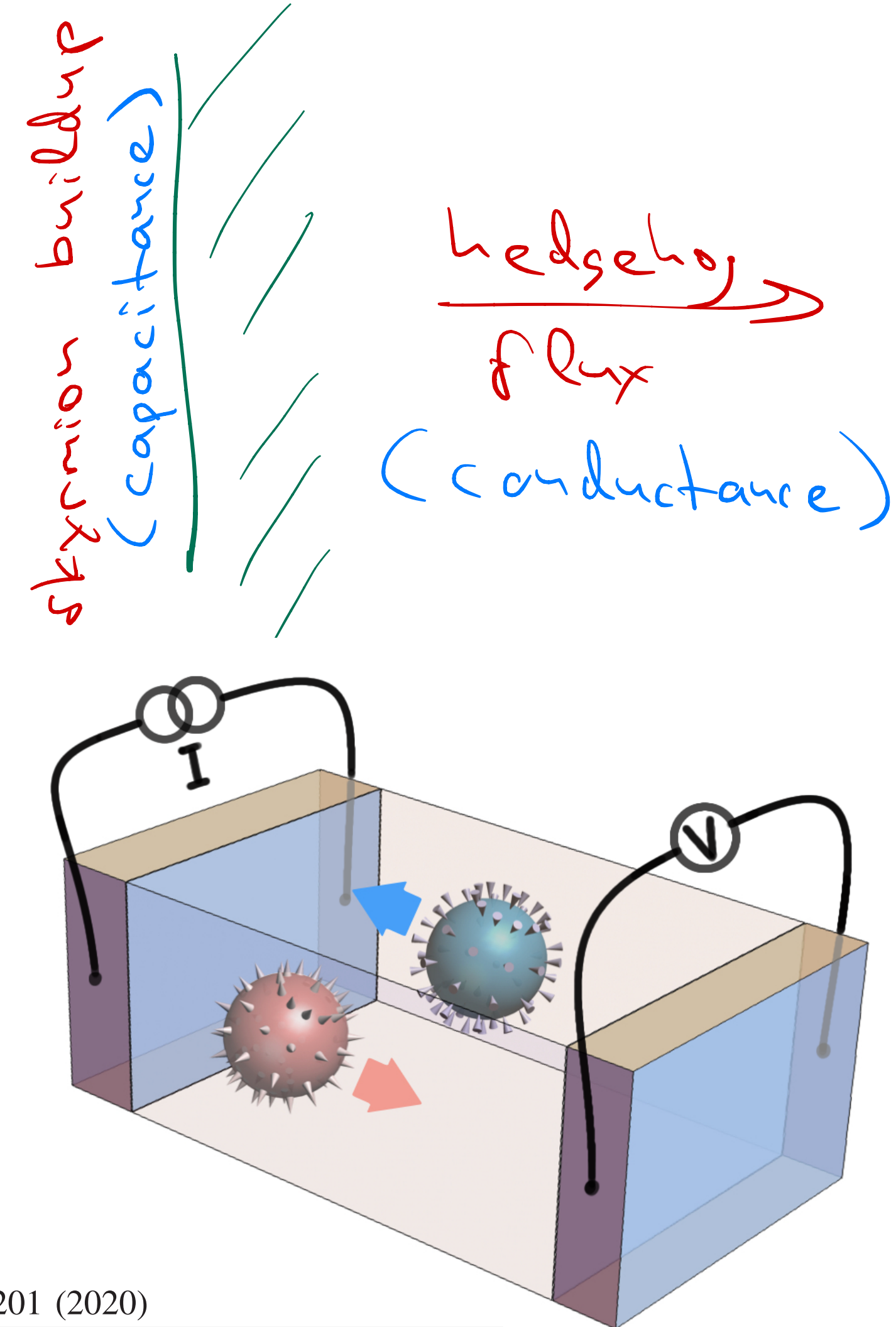
Engineering and imaging hedgehog metamaterials



Creation and observation of topological magnetic monopoles and their interactions in a ferromagnetic metalattice
Soft X-ray imaging by Miao's group at UCLA [Rana et al., *Nature Nano.* (2023)]

Extending to 3D bulk: Hedgehog flow

The 3D hedgehog flow with skyrmionic bulk/boundary correspondence is closely analogous to the 2D vorticity flow with winding bulk/boundary correspondence



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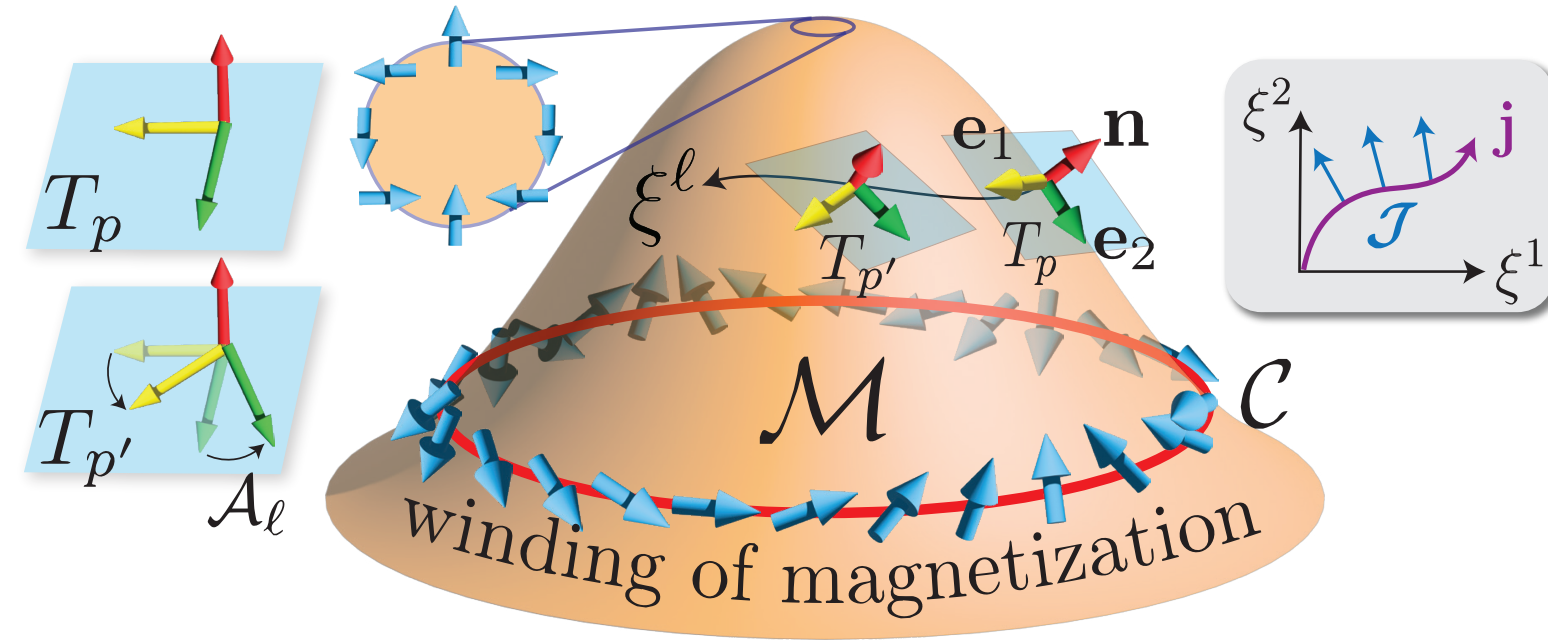
Editors' Suggestion

Topological Transport of Deconfined Hedgehogs in Magnets

Ji Zou¹, Shu Zhang,^{*} and Yaroslav Tserkovnyak

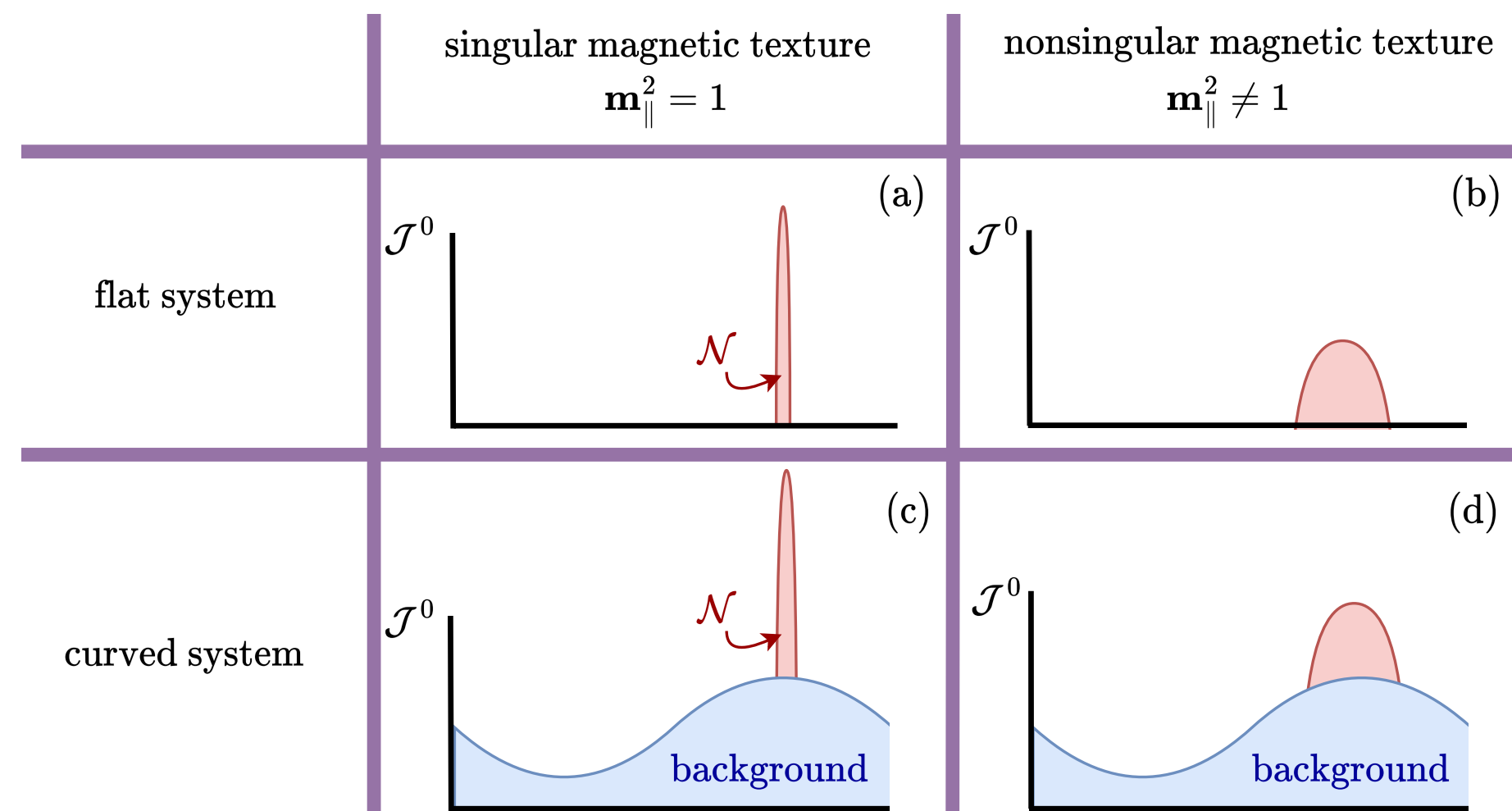
Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

Hydrodynamics on curved surfaces



- Net topological charge is defined in terms of a winding 1-form (the density is constructed via exterior derivative, utilizing generalized Stokes' theorem):

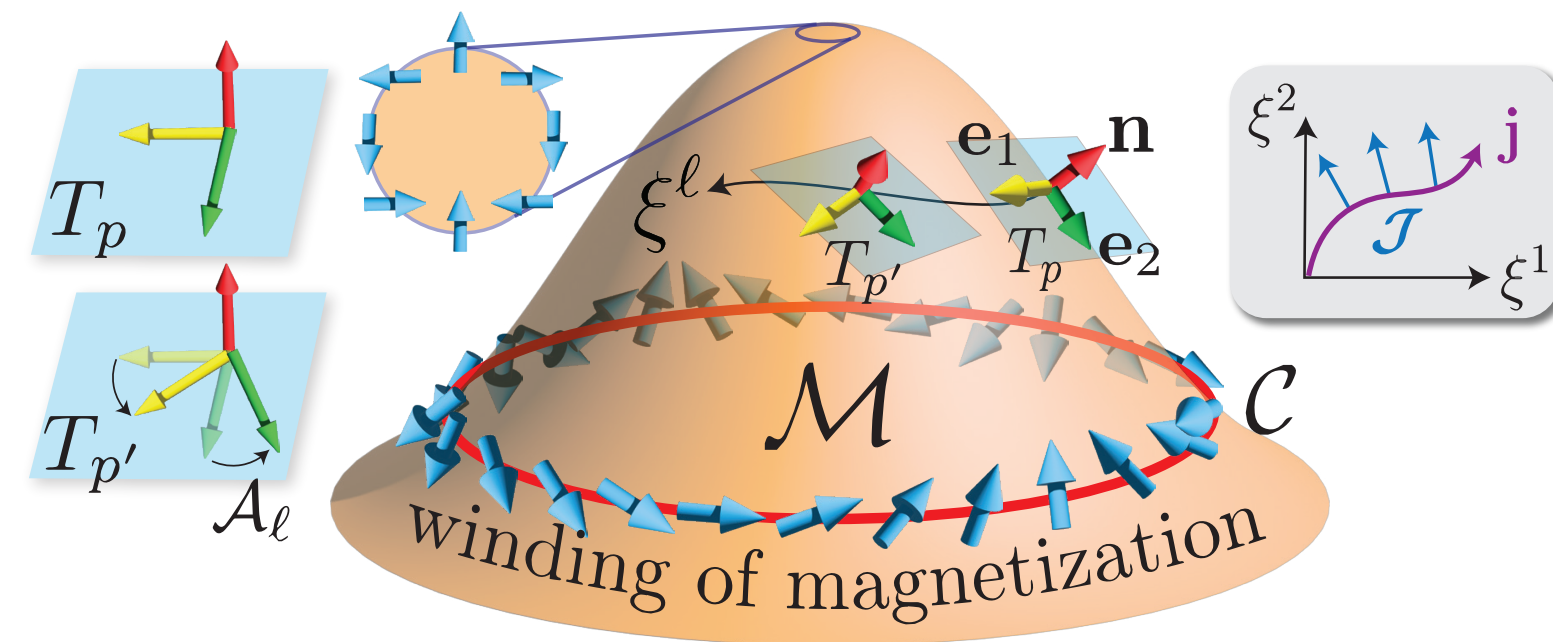
$$Q = \int_{\partial S} \rho_w = \frac{1}{2\pi} \int_{\partial S} d\xi^i \mathbf{m}_{\parallel}^2 D_i \varphi \quad \text{where} \quad D_i \varphi \equiv \partial_i \varphi - \mathbf{e}_1 \cdot \partial_i \mathbf{e}_2$$



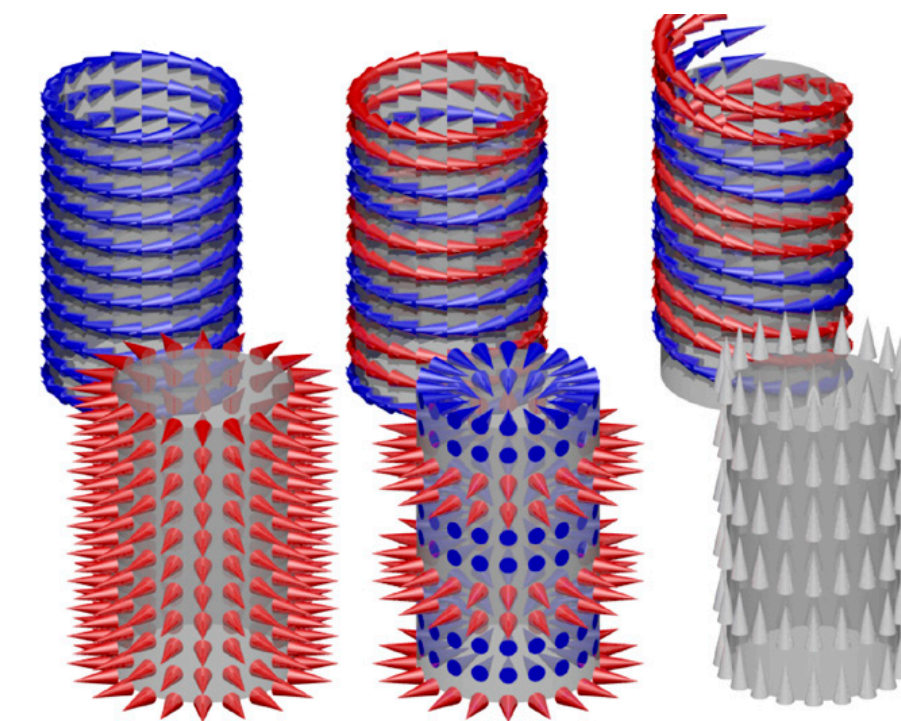
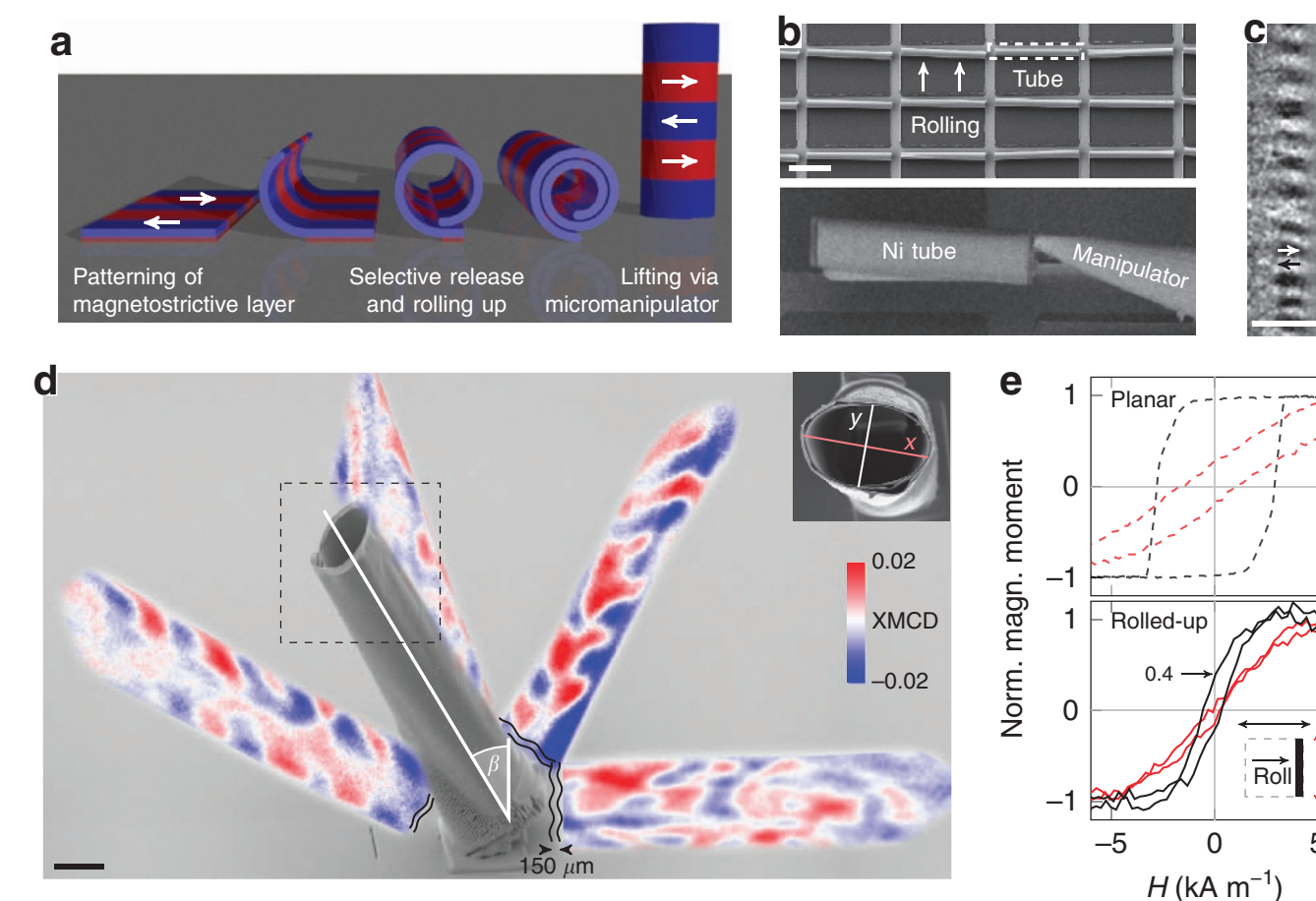
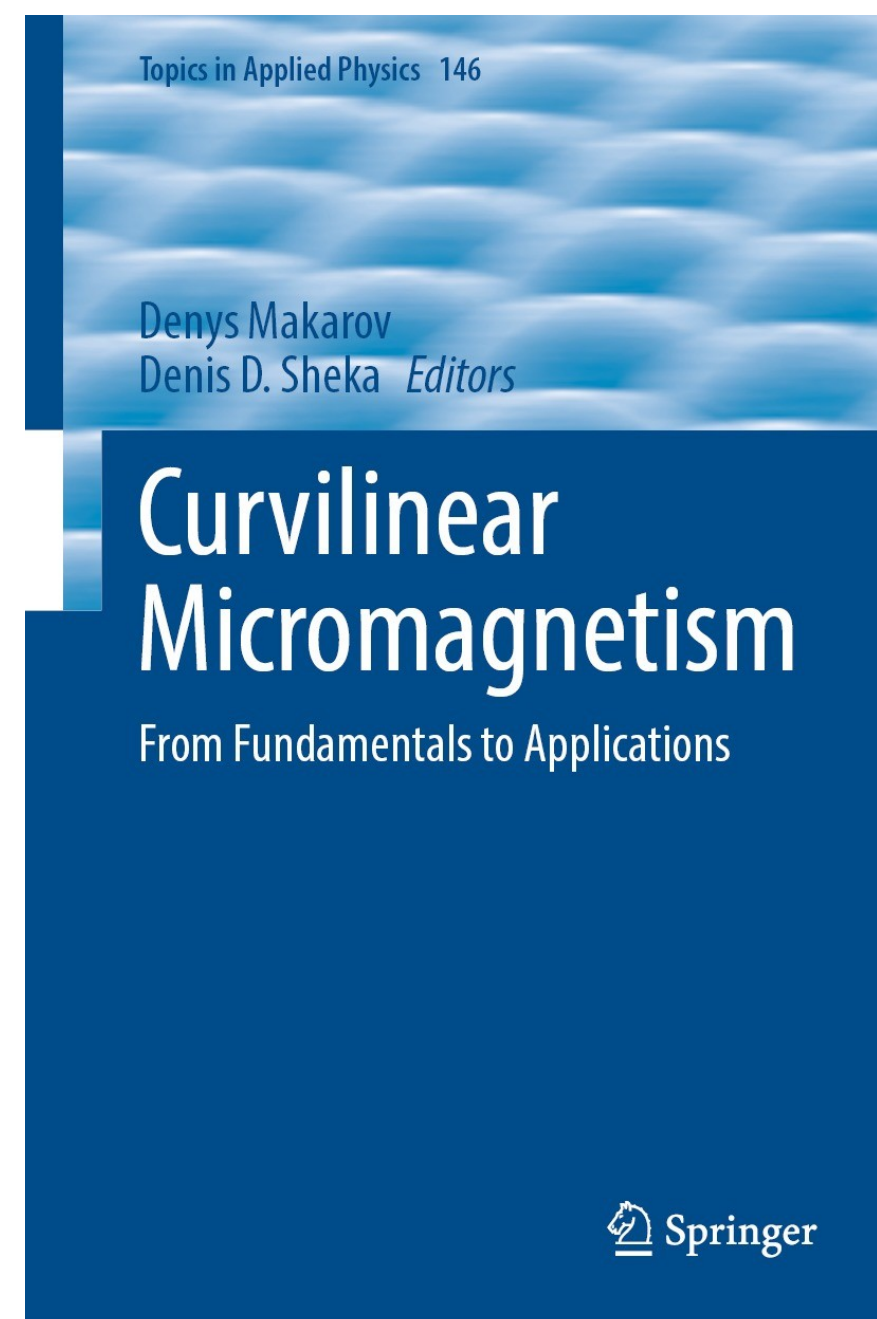
in the strongly easy-plane limit:

$$Q = \mathcal{N} - \frac{1}{2\pi} \int_S d\xi^1 d\xi^2 \sqrt{g} \mathcal{K} \quad (\text{Mermin-Ho})$$

Transport of vorticity on curved surfaces

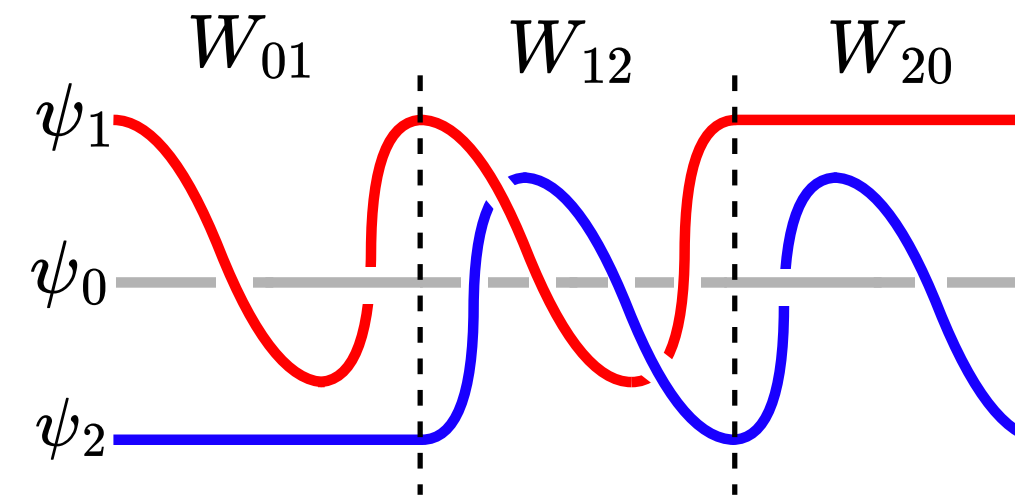


- Can nontrivial geometry reduce the symmetries enough to allow to drive topological hydrodynamics even in a single magnetic layer?



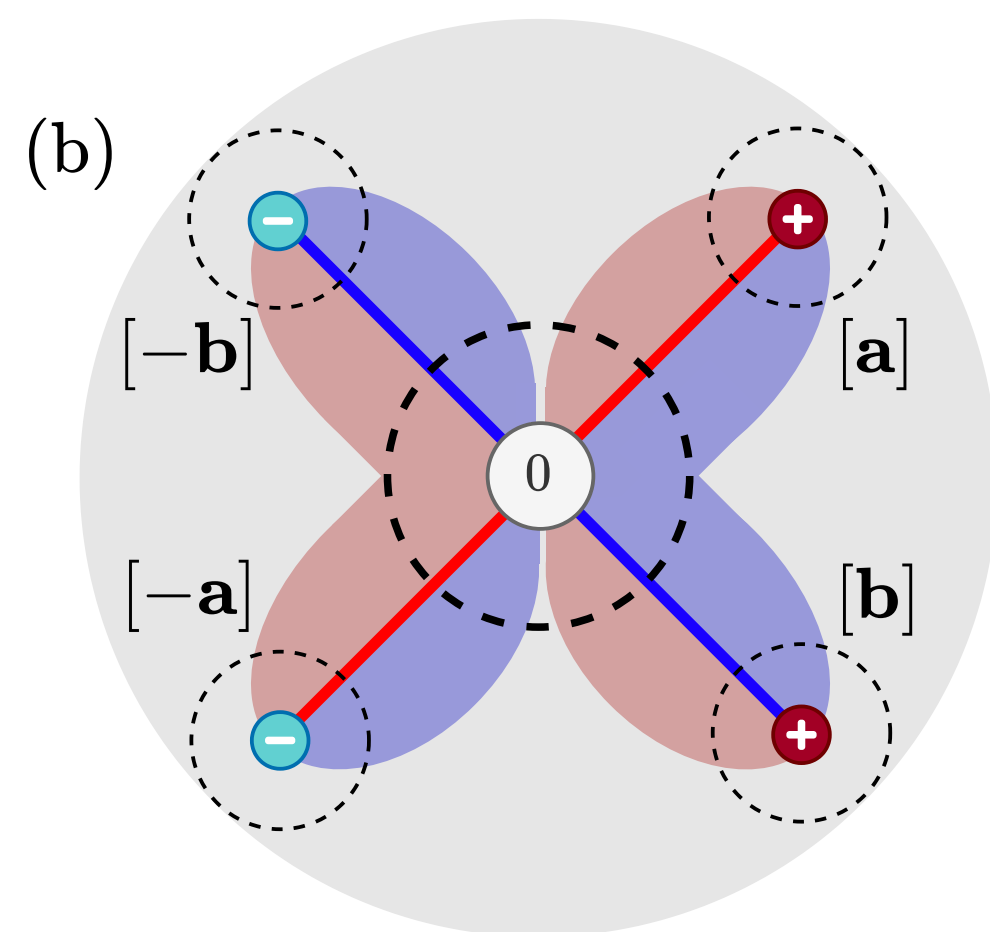
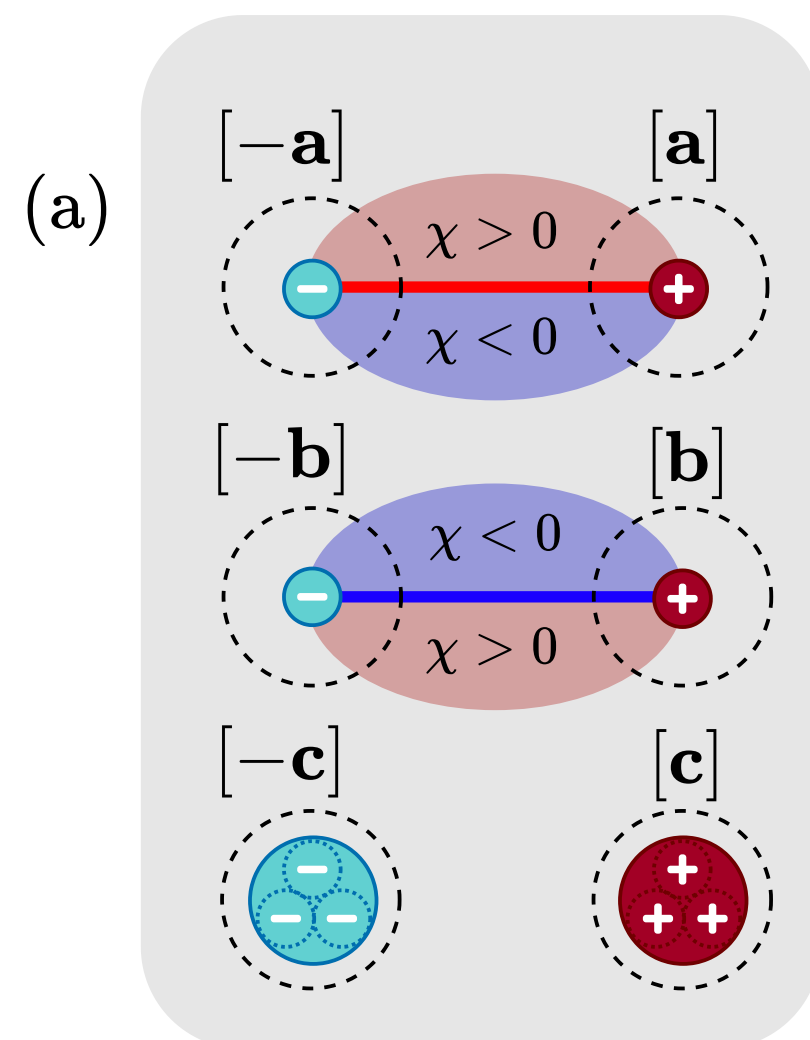
Non-Abelian homotopy group requires revisiting basic notions of hydrodynamics

We study a model based on two repulsive superfluid condensates $\psi_1, \psi_2 \in \mathbb{C}$



Low-energy theory realizes rich structure of pointlike defects:

- the order-parameter space is given by the *ordered configuration space* of three *c*-numbers: $C_3(\mathbb{C})$
- the fundamental group is given by the (non-Abelian) *pure braid group*: $\pi_1(C_3) = P_3$



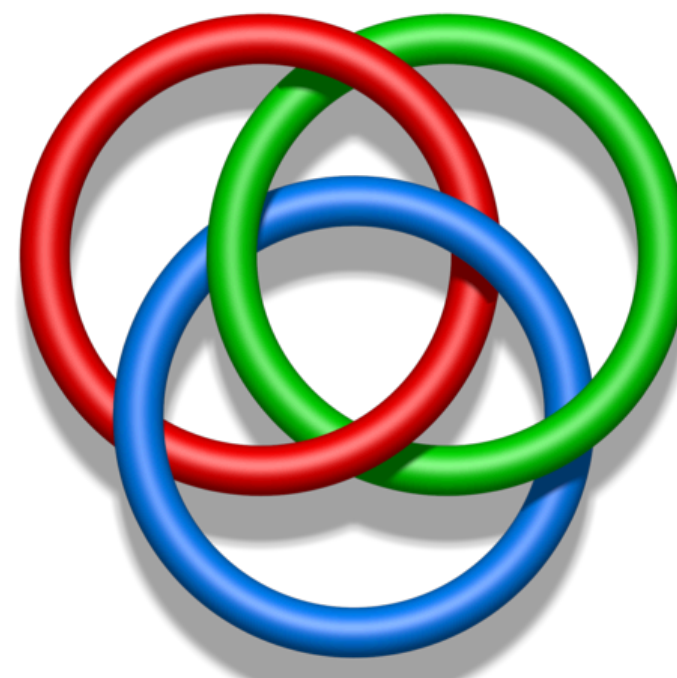
$$P_3/Z(P_3) = F_2 = \langle a, b \rangle$$

free group

center
(overall winding)

1st condensate winding

2nd condensate winding



“Borromean” vortex: each individual net winding W_{ij} is zero (yet “second-order” winding is 1)

Generalizations and broader themes

- **Going beyond superfluid-like planar winding**
 - *Dimensionality-dependent generalizations: vortices, baby (2D) skyrmions, hedgehogs, Shankar (3D) skyrmions, hopfions etc.*
- **Classifications based on topology vs differential geometry**
 - *Conservation laws based on topological invariance (absolute vs relative homotopies) vs generalized Stokes' theorems*
- **Curved geometries as a structural handle on symmetries**
 - *Do the conservation laws survive nontrivial shapes? Can we exploit it?*
- **What about thermal and quantum fluctuations?**
 - *Local vs nonlocal topological flavors, quantum spin-lattice constructions and field theory*
- **Non-Abelian topological flavors**
 - *Can we nucleate and transport topological charges that cannot be counted?*