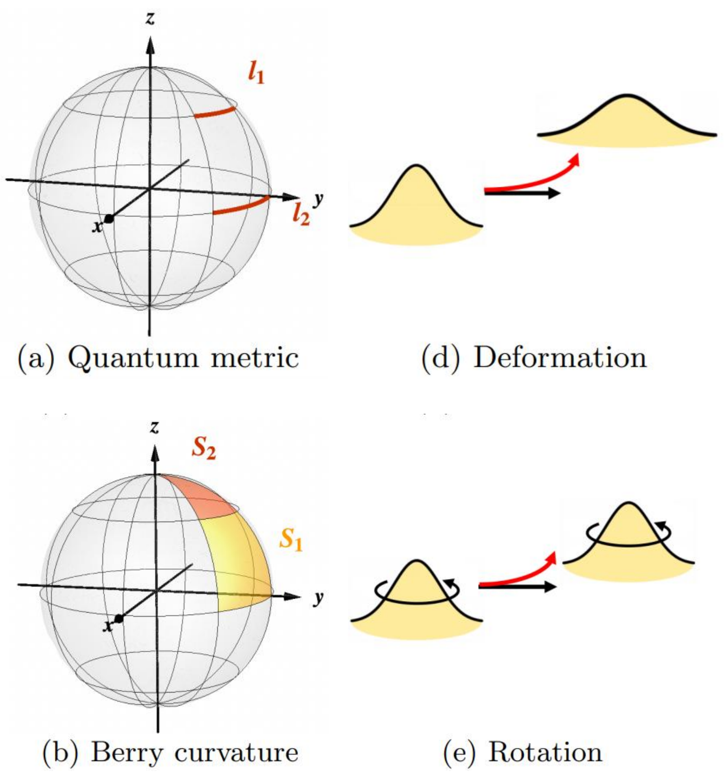
Quantum Geometry in Magnetic Quantum Materials

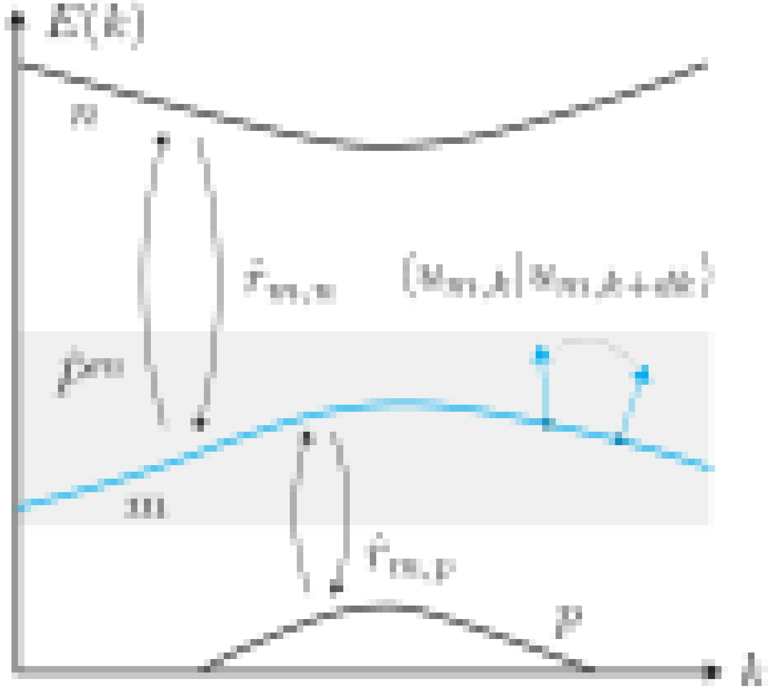


Binghai Yan

Pennsylvania State University

2026 Principles and Applications of Symmetry in Magnetism Summer School
Colorado State University, Jul 16,2026

Quantum geometry



$$\boxed{Q_n^{ab} = \sum_{m \neq n} r_{nm}^a r_{mn}^b = g_n^{ab} - \frac{i}{2}\Omega_n^{ab}.}$$

$$r_{nm}^a(\mathbf{k}) = \langle u_{n,\mathbf{k}}|\, i\partial_{\mathbf{k}_a}\, |u_{m,\mathbf{k}}\rangle, \qquad n \neq m$$

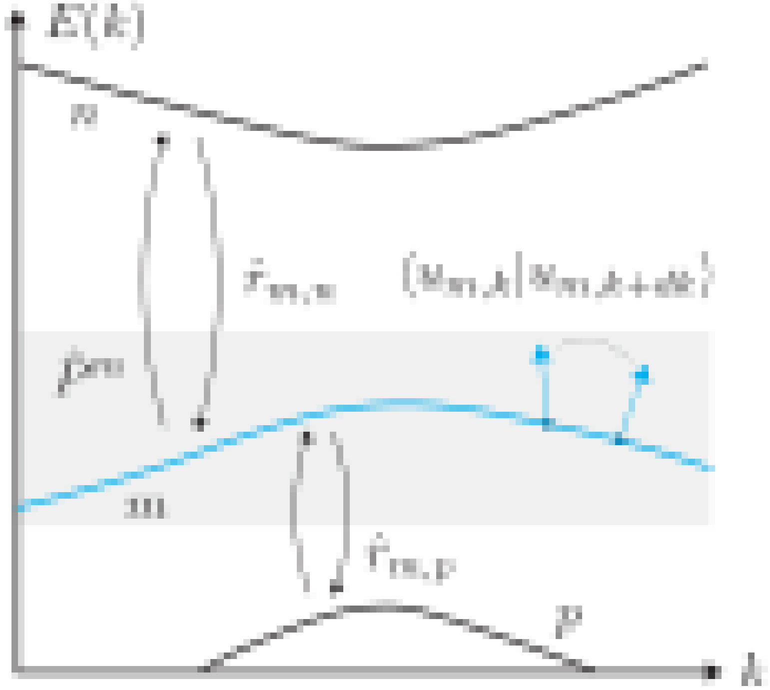
$$\mathcal{A}_\mu^{(m)} = i\langle u_m|\partial_{k_\mu} u_m\rangle$$

$$\langle u_{\mathbf{k}}|u_{\mathbf{k}+d\mathbf{k}}\rangle = 1 + \langle u|\partial_\mu u\rangle dk_\mu = 1 - i\mathcal{A}_\mu dk_\mu \simeq e^{-i\mathcal{A}_\mu dk_\mu}$$

Phase change

$$\Omega = \nabla \times A$$

Quantum geometry



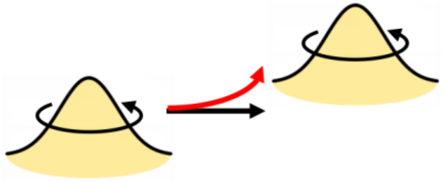
$$\boxed{\left|\langle u_{m,\mathbf{k}}|u_{m,\mathbf{k}+d\mathbf{k}}\rangle\right|^2 = 1 - g^{(m)}_{\mu\nu}\, dk_\mu dk_\nu + O(dk^3)}$$

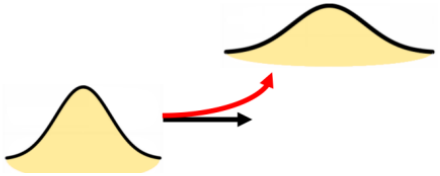
Amplitude change

Quantum Geometry

Quantum geometric tensor

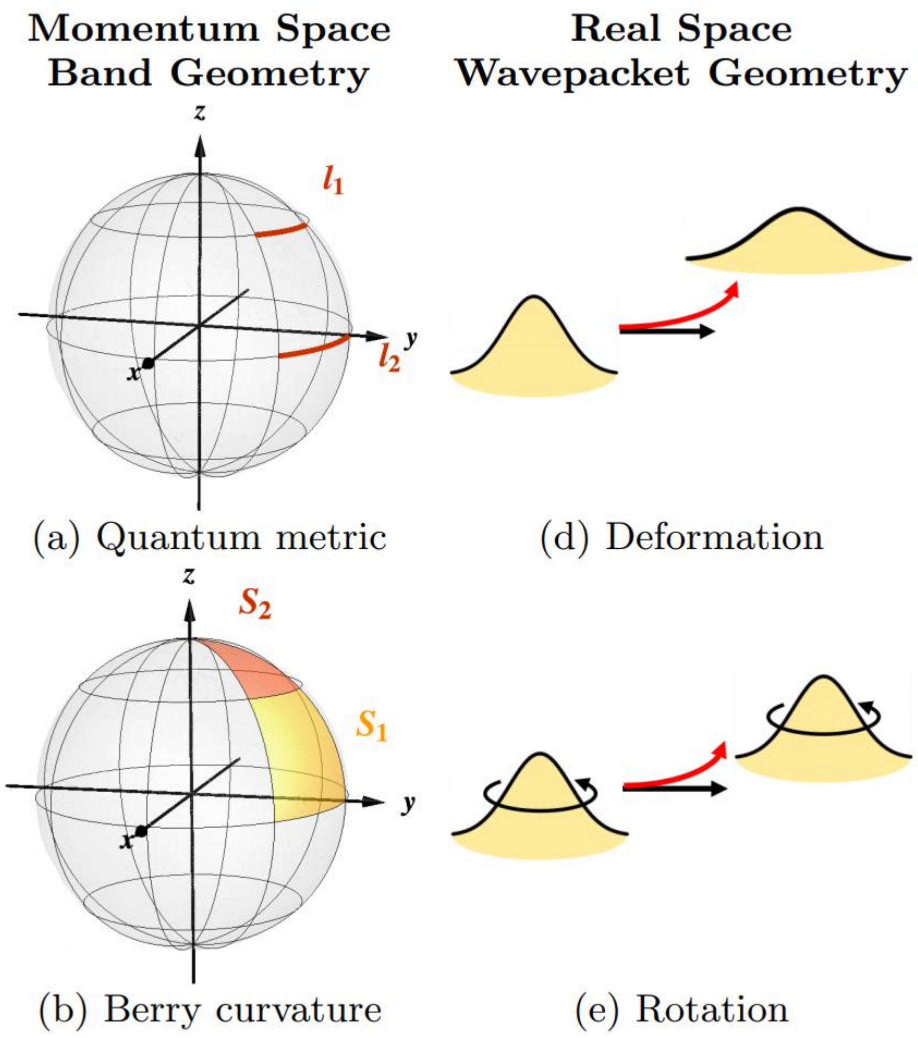
		Consequences in transport
Berry curvature	$\boldsymbol{\Omega}_{\mu\nu}$	Anomalous Hall Effect (AHE) QAHE, Nonlinear AHE
Quantum metric	$\boldsymbol{g}_{\mu\nu}$ $\quad < r^2 >$	Theory for FCI, SC etc. **Experimental detection?**





$$\boxed{Q_n^{ab} = \sum_{m \neq n} r_{nm}^a r_{mn}^b = g_n^{ab} - \frac{i}{2}\Omega_n^{ab}.}$$

Quantum geometry and nonlinear responses

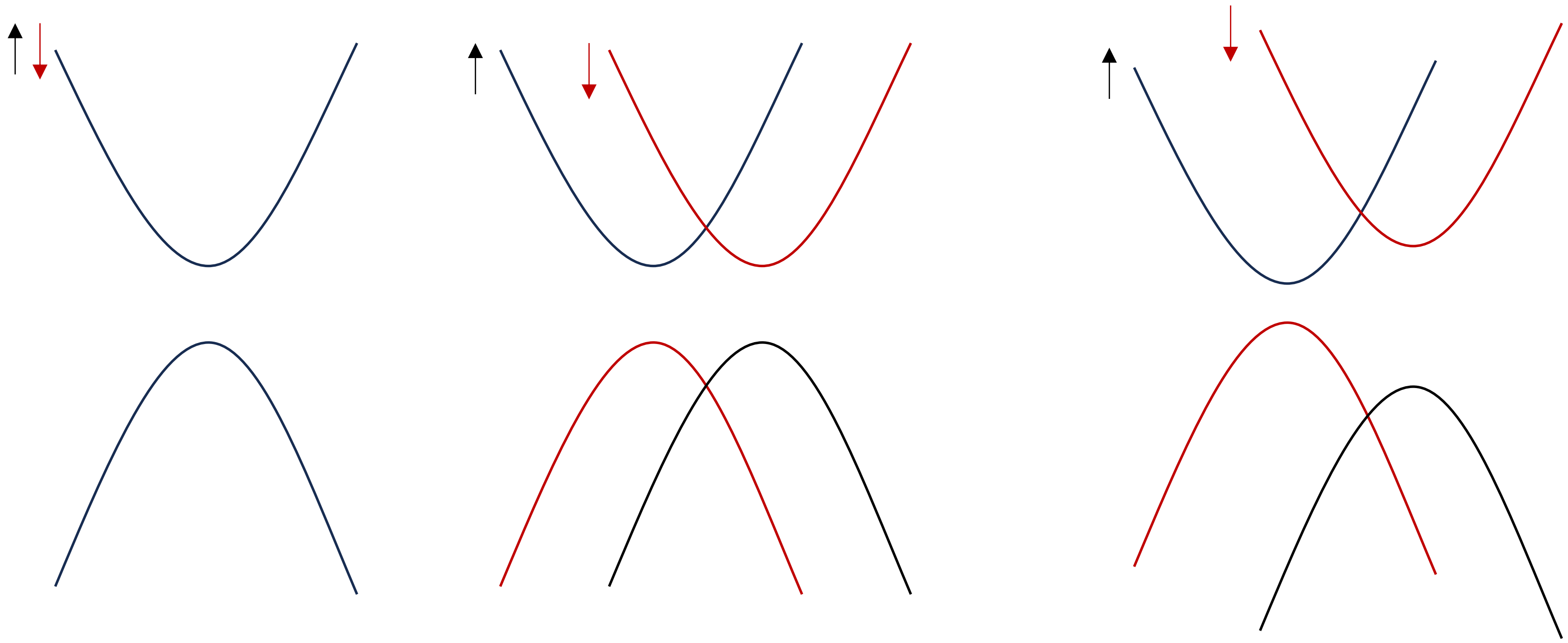


Revealing Quantum Geometry in Nonlinear Quantum Materials

Yiyang Jiang,[1,2] Tobias Holder,[3,*] and Binghai Yan[1,2,†]

Report of Progress in Physics, 88 076502 (2025)
[arXiv:2503.04943](https://arxiv.org/abs/2503.04943)

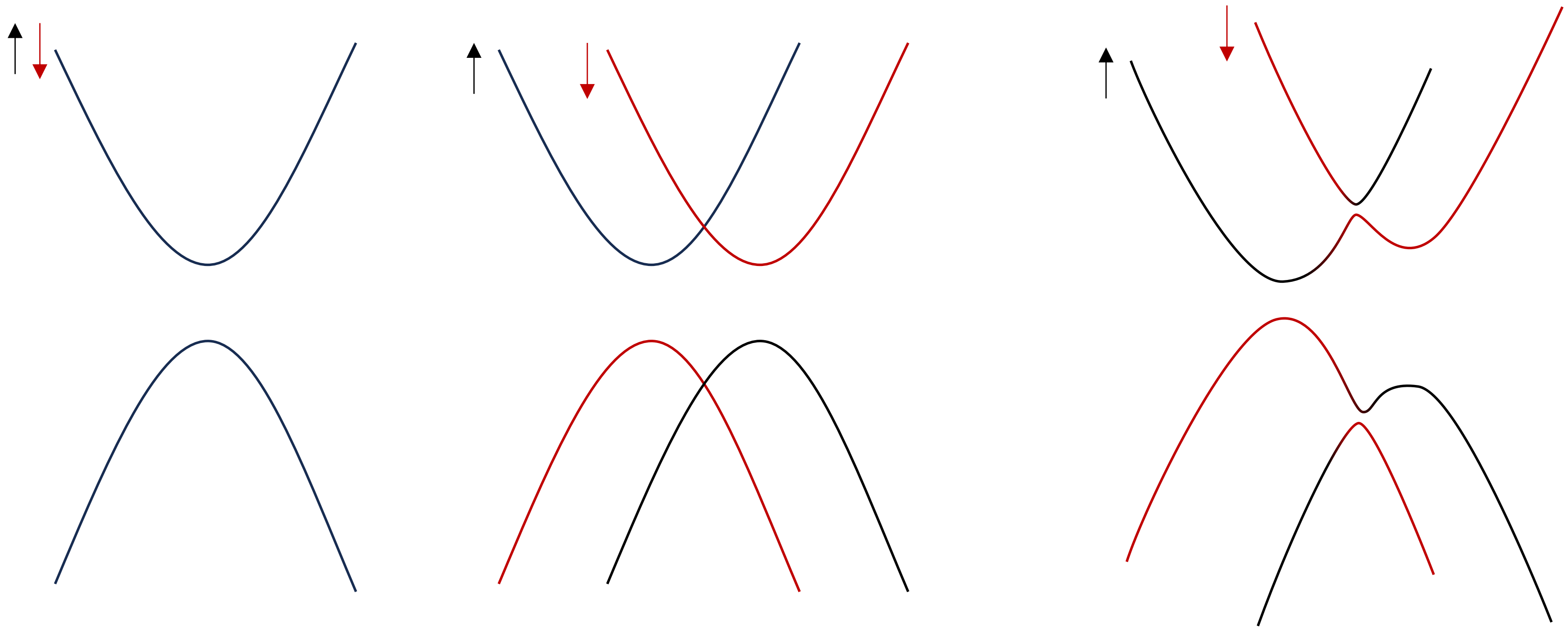
Symmetry breaking in solids



Inversion-breaking

Inversion-breaking
Time-reversal-breaking

Symmetry breaking in solids



Inversion-breaking

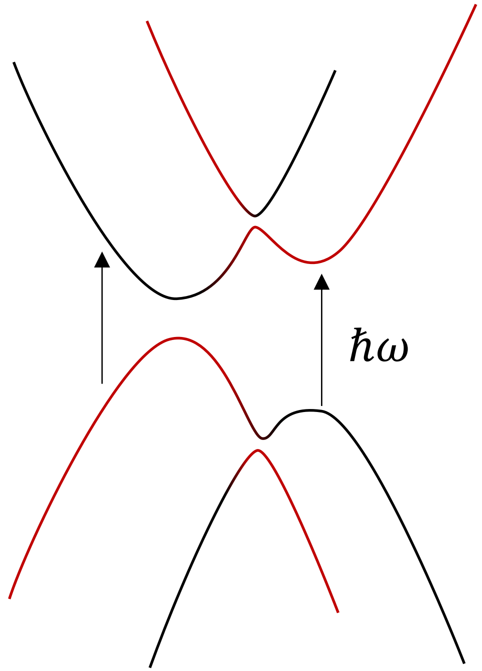
Inversion-breaking
Time-reversal-breaking

Symmetry breaking in solids

New injection current

$$J \sim \left(\sum \boldsymbol{g}_{\boldsymbol{nm}}^{\boldsymbol{aa}} (v_n^c - v_m^c)\tau\right) \; E(\omega)^2$$

$$\boldsymbol{g}_{\boldsymbol{nm}}^{\boldsymbol{aa}} = |\boldsymbol{r}_{\boldsymbol{nm}}^{\boldsymbol{a}}|^2$$

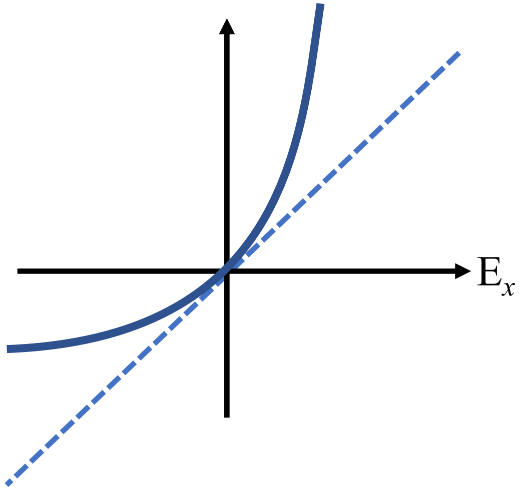


Y. Zhang et al. Nature Comm. (2019)
T. Holder, D. Kaplan, B. Yan PRR 2020

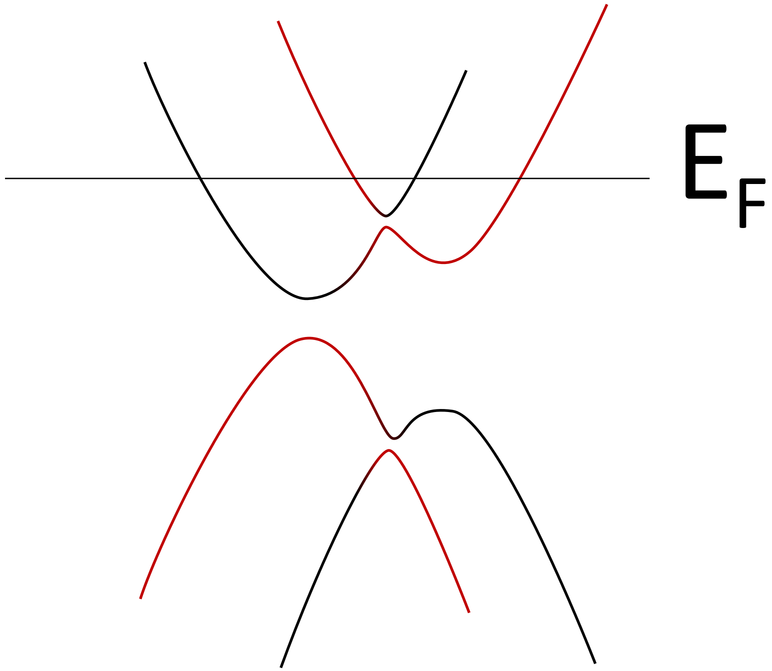
Inversion-breaking
Time-reversal-breaking

Symmetry breaking in solids

$$J_x = \sigma^{(1)} E_x + \sigma^{(2)} E_x{}^2$$



Nonreciprocal resistance
(Diode)

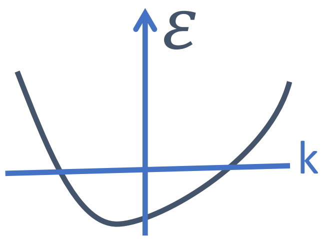


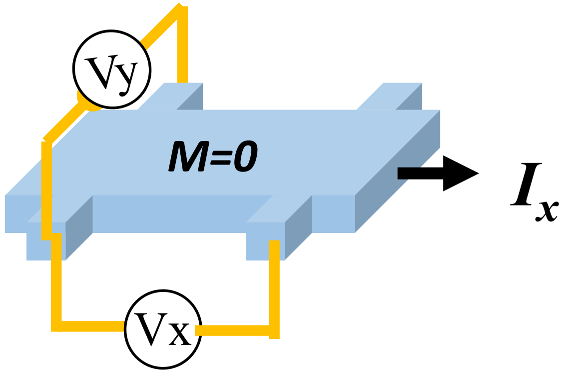
Inversion-breaking
Time-reversal-breaking

Nonlinear conductivities in a metal

$$\frac{j^c}{E^a E^b} \equiv \sigma^{ab;c} =$$
$$-\frac{e^3\tau^2}{\hbar^3}\sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\varepsilon_n$$ Drude
$$-\frac{e^3\tau}{\hbar^2}\sum_n \int_{\boldsymbol{k}} f_n(\partial_{k^a}\Omega_n^{bc} + \partial_{k^b}\Omega_n^{ac})$$ Berry curvature dipole
$$-\frac{e^3}{\hbar}\sum_n \int_{\boldsymbol{k}} f_n\left(2\partial_{k^c}G_n^{ab} - \frac{1}{2}\left(\partial_{k^a}G_n^{bc} + \partial_{k^b}G_n^{ac}\right)\right)$$ Quantum metric dipole

P- and T-Breaking

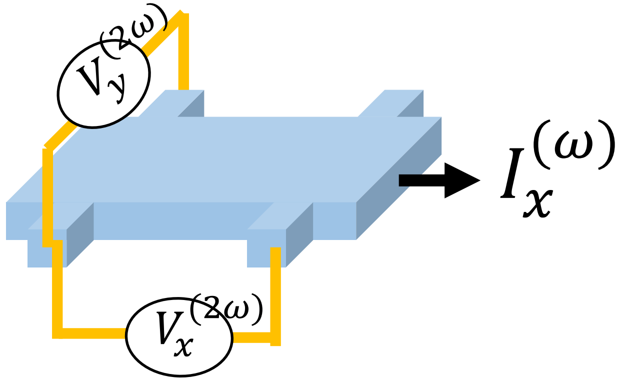




I. Sodemann and L. Fu, PRL (2015)

D. Kaplan. T. Holder, B. Yan, SciPost Physics 14, 082 (2023)
D. Kaplan. T. Holder, B. Yan, PRL 132, 026301 (2024)

Nonlinear conductivity in a metal



$$-\frac{e^3}{\hbar}\sum_n \int_{\boldsymbol{k}} f_n \left(2\partial_{k^c} G_n^{ab} - \frac{1}{2}\left(\partial_{k^a} G_n^{bc} + \partial_{k^b} G_n^{ac}\right)\right)$$

Band-normalized quantum metric $\quad G_n^{ab} = \dfrac{\{A_{nm}^a, A_{mn}^b\}}{2\varepsilon_{nm}}$

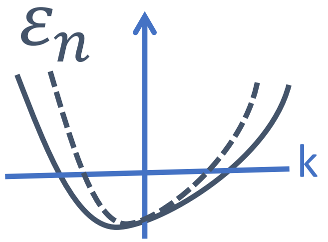
Pure Hall

$$\partial_{k^c} G_n^{ab} - \frac{1}{2}\left(\partial_{k^a} G_n^{bc} + \partial_{k^b} G_n^{ac}\right) \longrightarrow \Omega_n' \longrightarrow \tilde{A}_n^a = A_n^a + G_{nn}^{ab} E^b$$

Cf. Gao, Yang, Niu (2014)

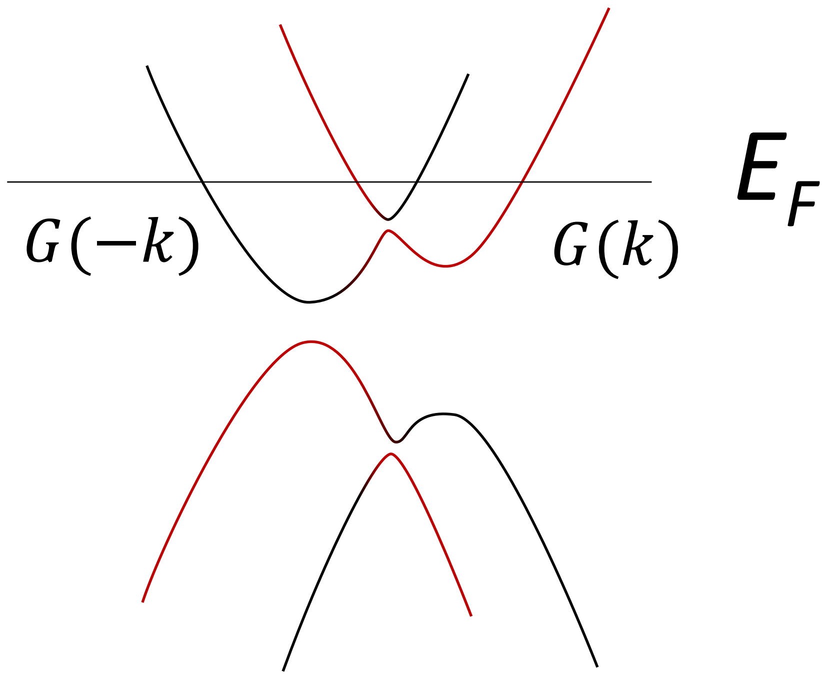
Longitudinal + Hall

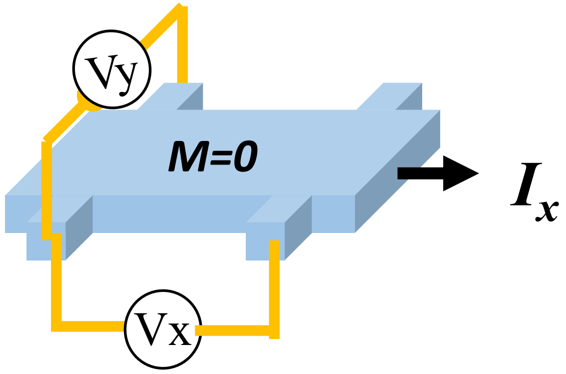
$$\partial_{k^c} G_n^{ab} \longrightarrow v_n' \longrightarrow \tilde{\varepsilon}_n = \varepsilon_n + e^2 G_n^{ab} E^a E^b$$



$$v_0 - \frac{e}{\hbar}\boldsymbol{E} \times (\boldsymbol{\Omega} + \boldsymbol{\Omega}') + v'$$

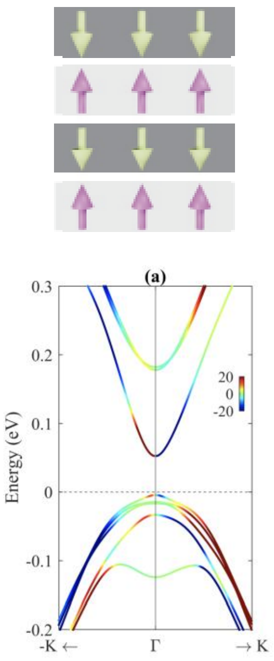
Quantum metric and nonlinear resistance

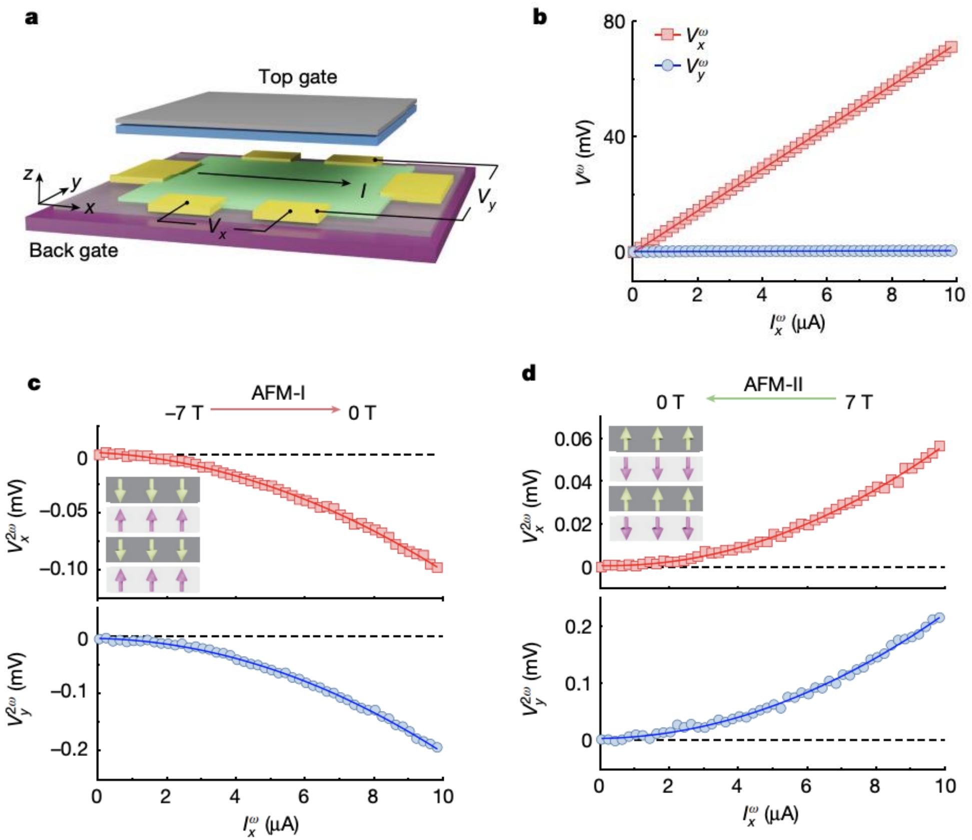




Inversion-breaking
Time-reversal-breaking

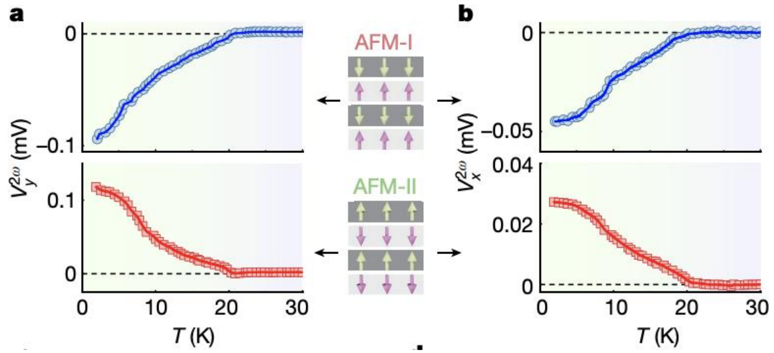
Experiment in $MnBi_2Te_4$ layers





Even layers of MnBi2Te4

- Quadratic resposne
- Revesed by swithcing the Neel order
- Appears only below T_N

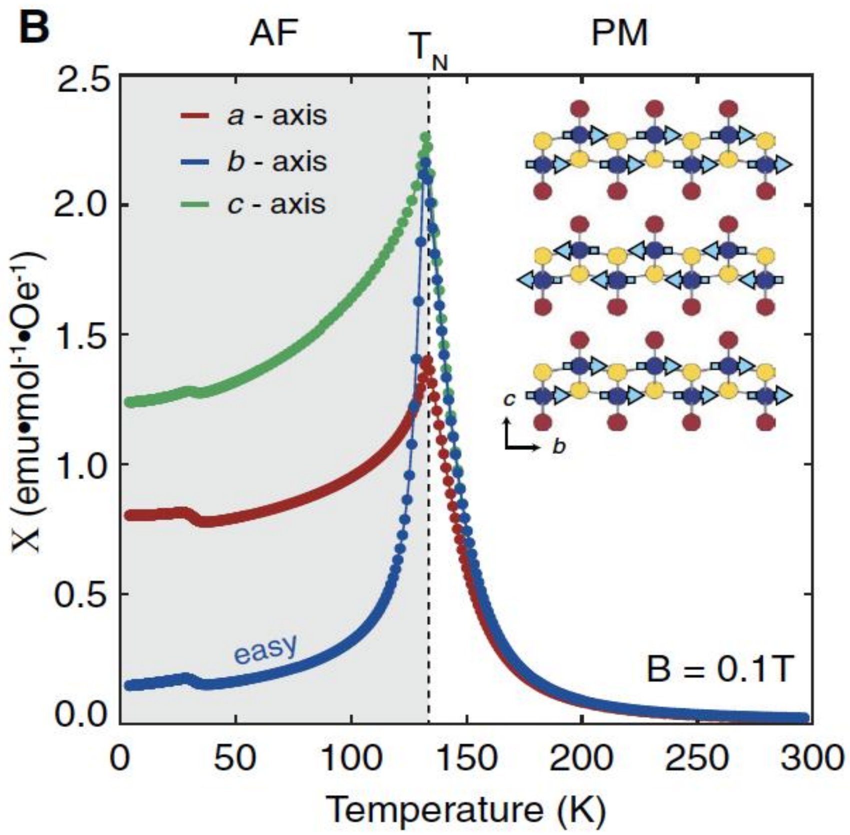


N. Wang et al, B. Yan, W. Gao, Nature **621**, 487 (2023)

See also S.Y. Xu, et al Science (2023)

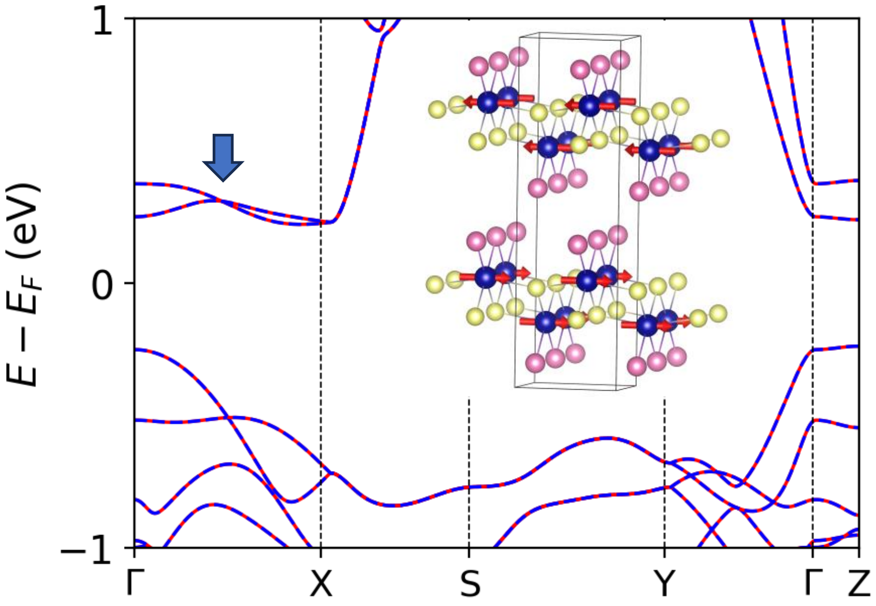
An AFM semiconductor CrSBr

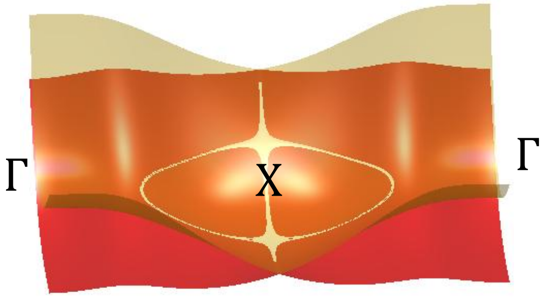
Parameters	$MnBi_2Te_4$	CrSBr
Neel temperature	25 K	132K
Magnetic axis	↑ ↑ ↓ ↓	← ← → →
Air stability	✗	✓



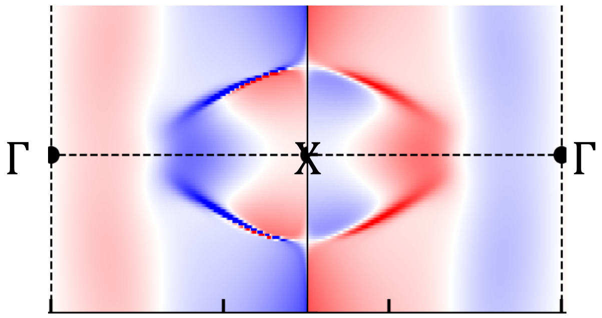
E. Telford et al, Adv. Mater., ***32***, 2003240 (2020)

Topological nodal lines and quantum metric





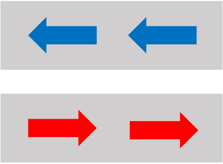
Nodal lines

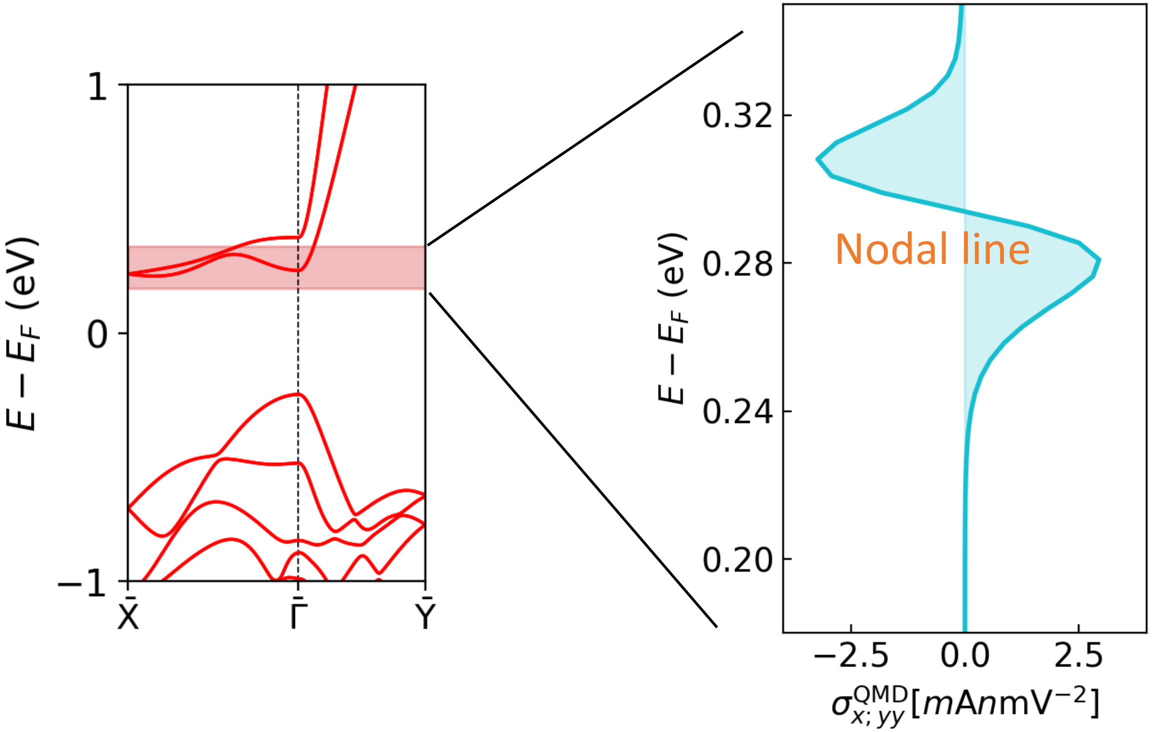


Quantum metric distribution

CrSBr is electron-doped in experiments.

Bilayer CrSBr

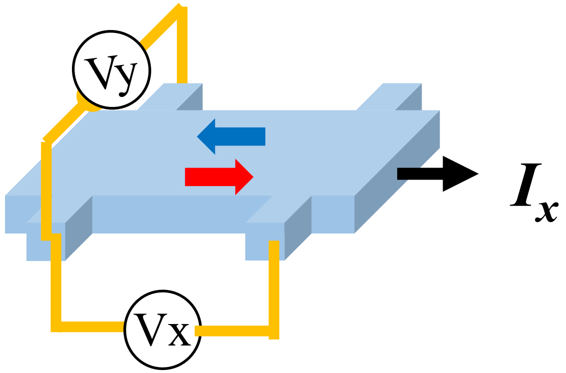




Nonlinear conductivity

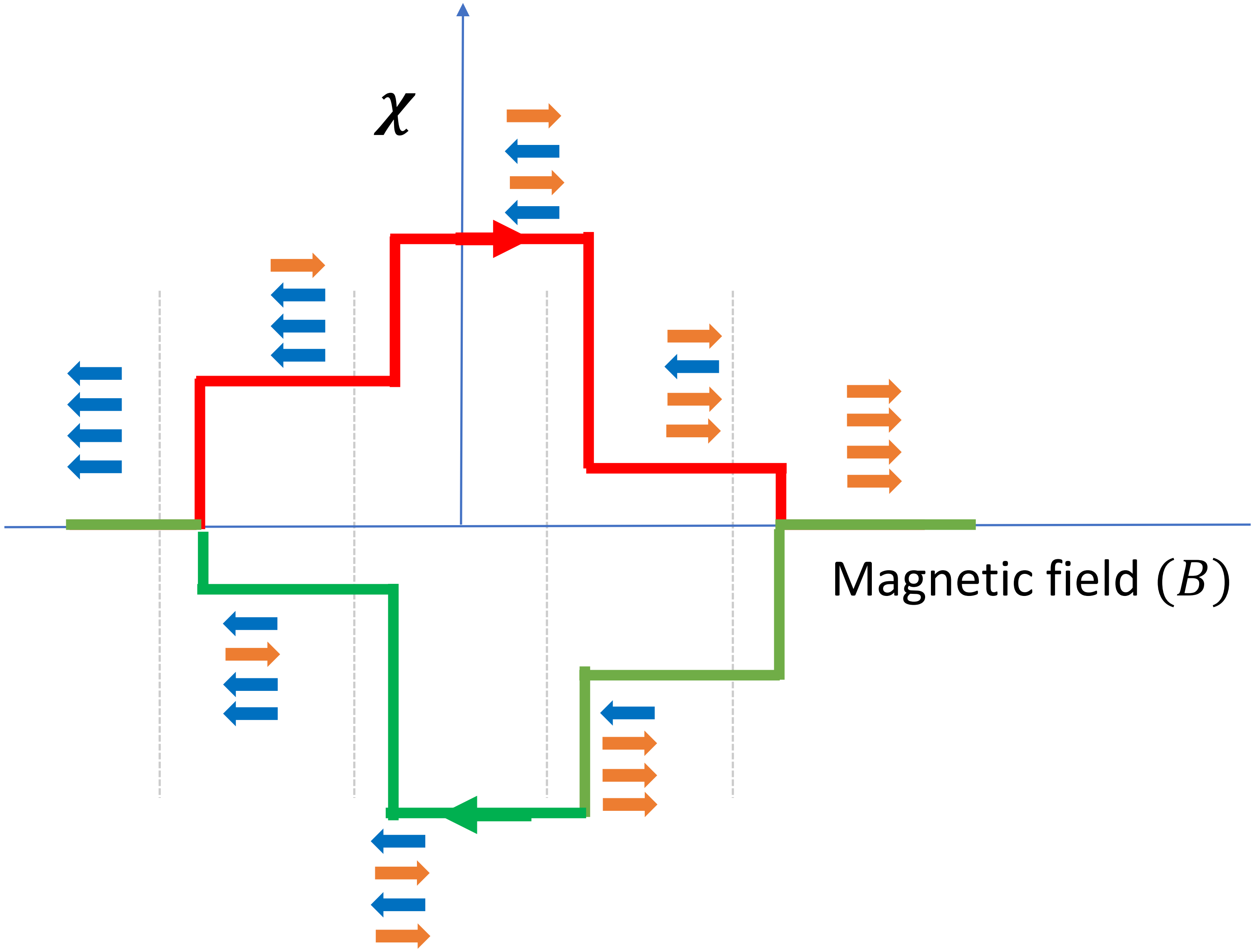
$$J_i^{2\omega} = \sigma_{i;jj} E_j^2$$

$$\sigma_{x;xx}, \sigma_{x;yy}$$

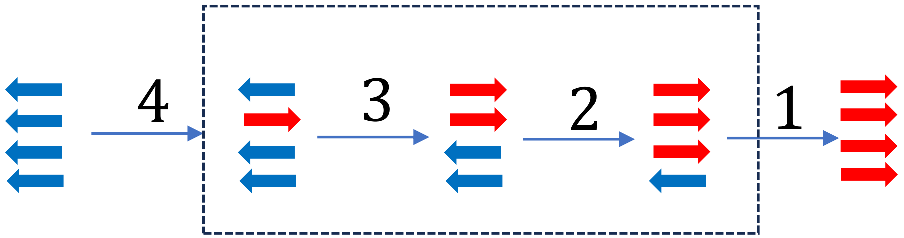


K. Das, Y. Zhao, **B. Yan**, Nano Lett. 25 91899196 (2025), arXiv: 2504.04526.

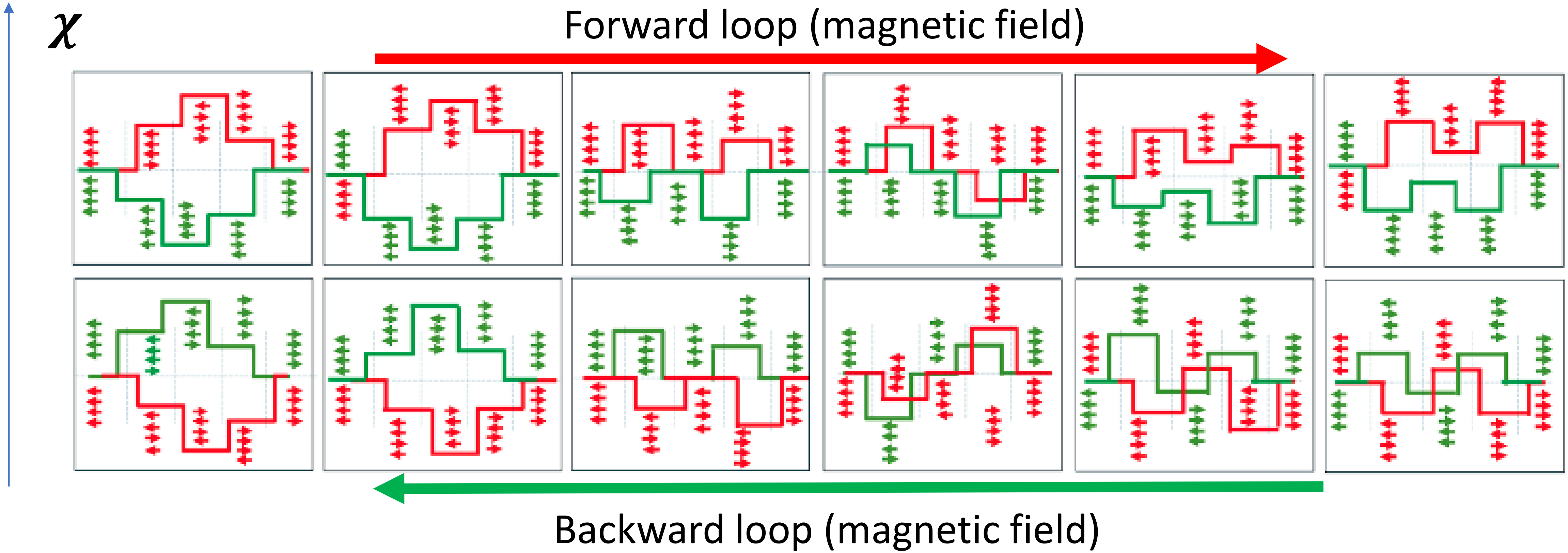
Reveal the spin order by nonlinear responses



All possible hysteresis loops of four layer



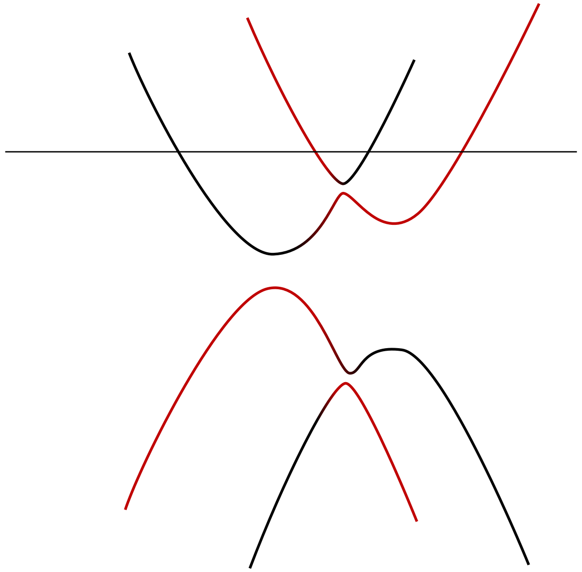
Total=24 loops

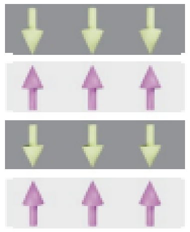


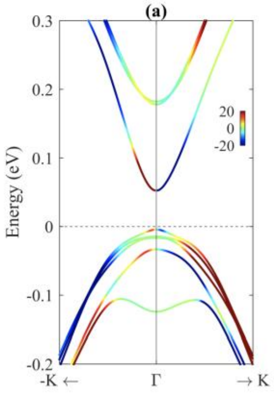
These loops reflect the underlying spin order

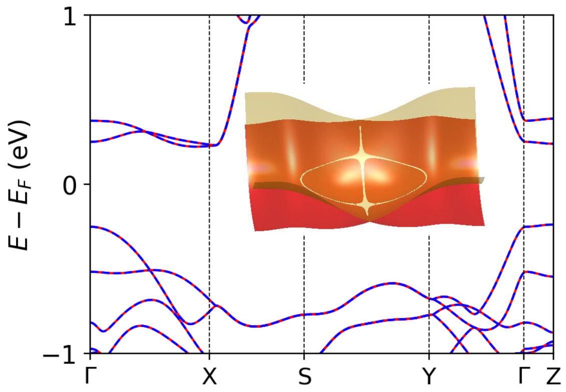
Topological bands enhance quantum geometry

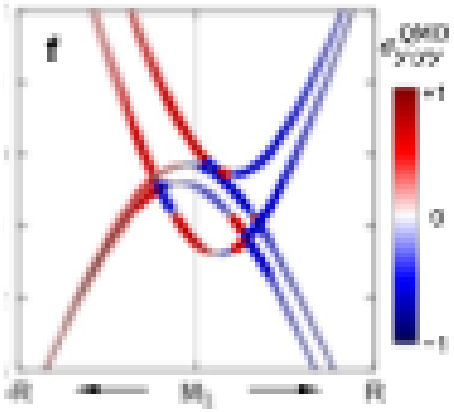
- Quantum metric, spin reversal
- Topology, band crossing / anti-crossing
- Symmetry-breaking, spin, structure





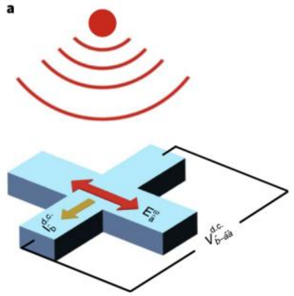


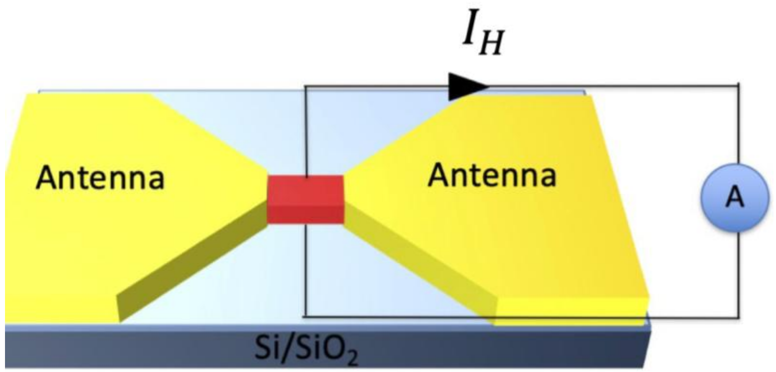




Technology impacts

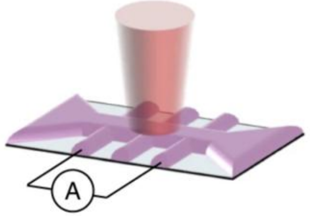
Energy harvesting, green
- Bulk PV effect
- RF rectification





Y.Zhang & L.Fu, PNAS 2021

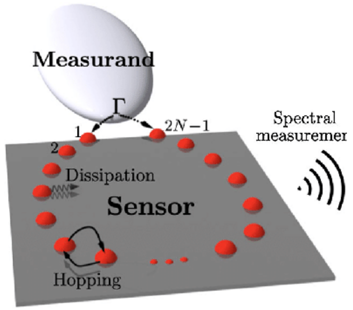
Photodetector, IR, THz



H. Yang et al. Nature Nano 2021

Non-Hermitian Topological Sensor

Entanglement photons



J. C. Budich and E. J. Bergholtz, Phys. Rev. Lett. (2020)

Fulga, van den Brink,et al. Nature Physics 20, 395 (2024)

Acknowledgement

Thanks for your attention!

My group

Tobias Holder
(Tel Aviv Uni. now)

Daniel Kaplan
(Central Florida Uni. now)

Yiyang Jiang

Yufei Zhao

Kamal Das

erc

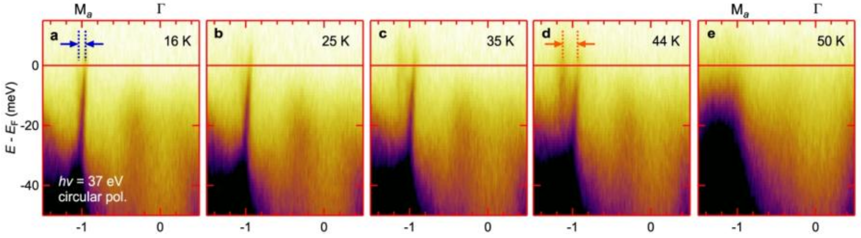
ISRAEL SCIENCE FOUNDATION

MRSEC NSF

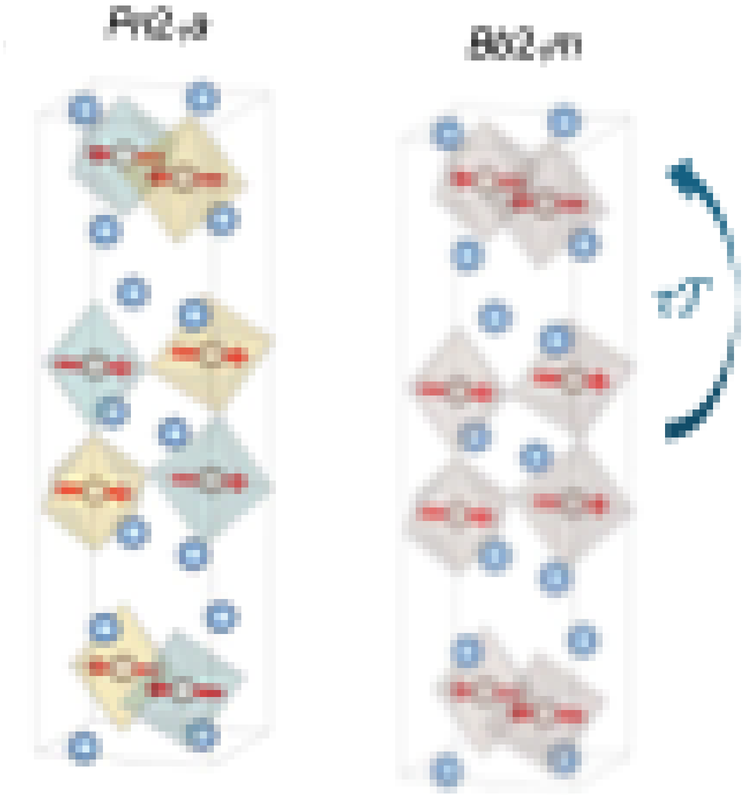
Hidden symmetry-breaking

Zhiqiang Mao
(PSU)

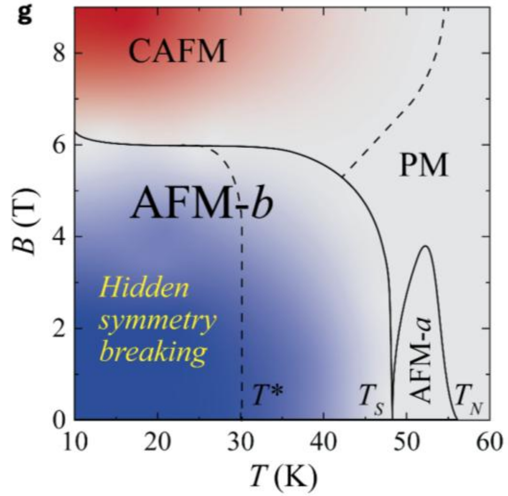
$Ca_3Ru_2O_7$, a correlated polar metal with phase transitions.

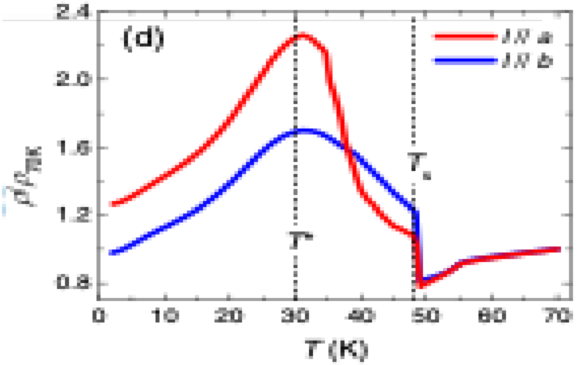


ARPES revealed Dirac like bands after transition

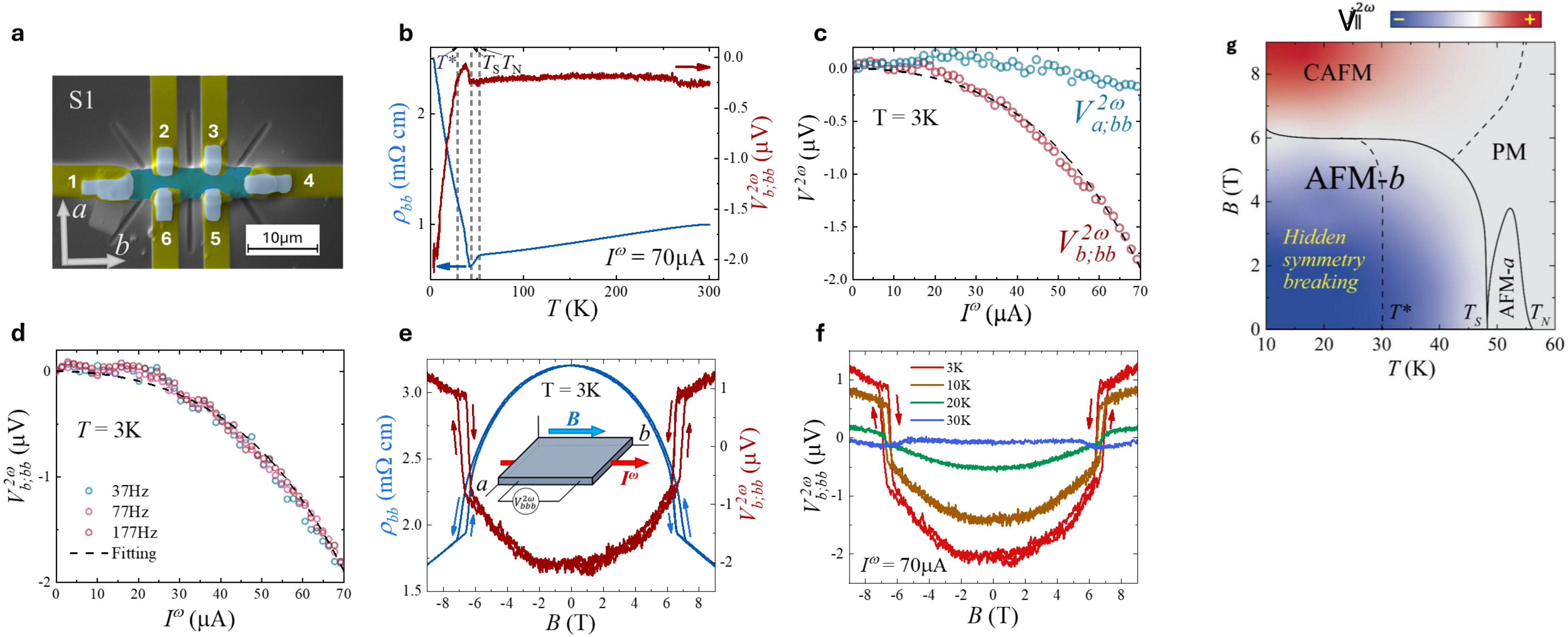


New structure (DFT):
Atomic displacement ~ 0.001 Å





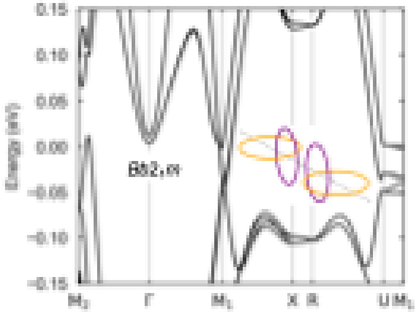
Nonlinear transport experiment

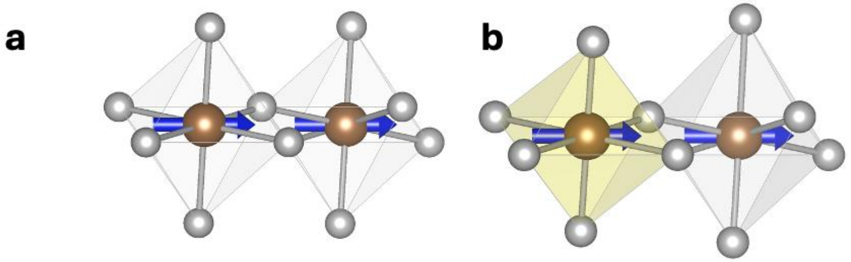


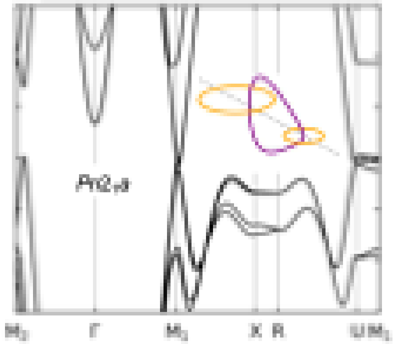
Mali et al. Nature Comm (2026), arXiv:2510.18144

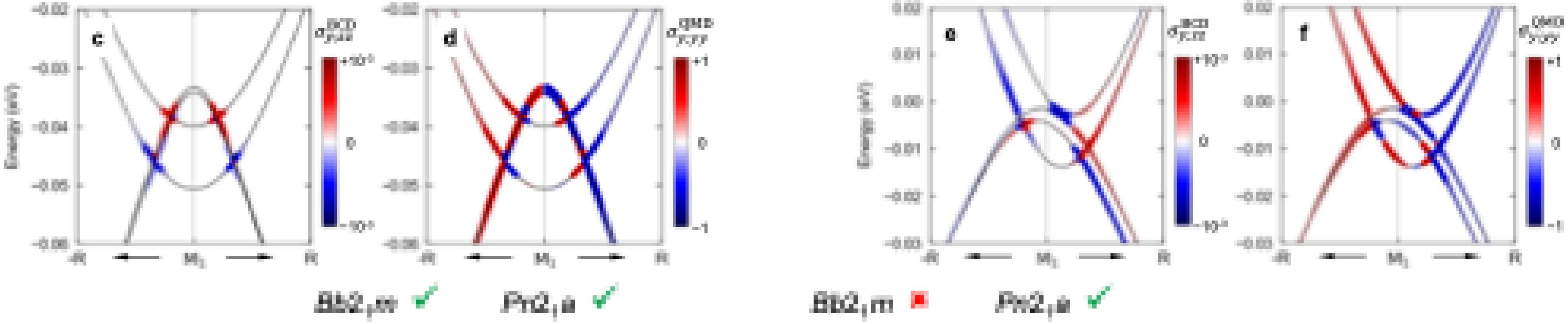
In creasing interaction (U)

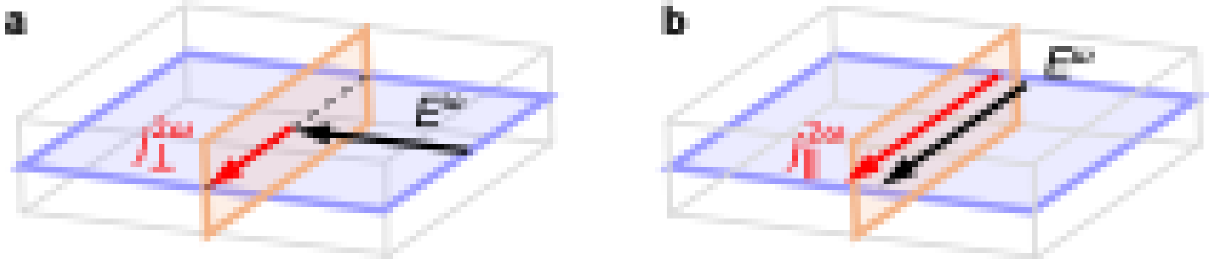
Symmetry-breaking











$$\boldsymbol{\Omega}_{\mu\nu} = i[A_\mu, A_\nu]$$

$$\boldsymbol{G}_{\mu\nu} = \frac{1}{2}\{A_\mu, A_\nu\}/\varepsilon$$

Berry curvature dipole << Quantum metric dipole

Y. Zhao, Z. Mao, BY, Phys. Rev. B 112, 165127 (2025), arXiv:2505.20603

Altermagnet -- $Ca_3Ru_2O_7$