Quantum Geometry in Magnetic Quantum Materials

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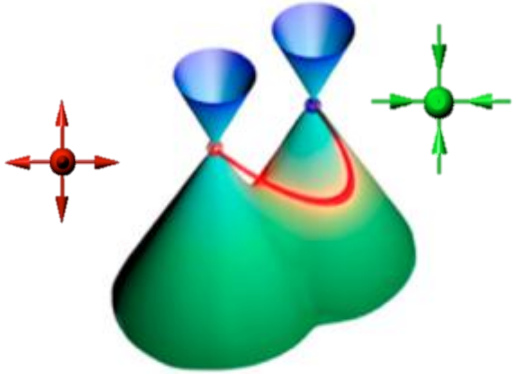
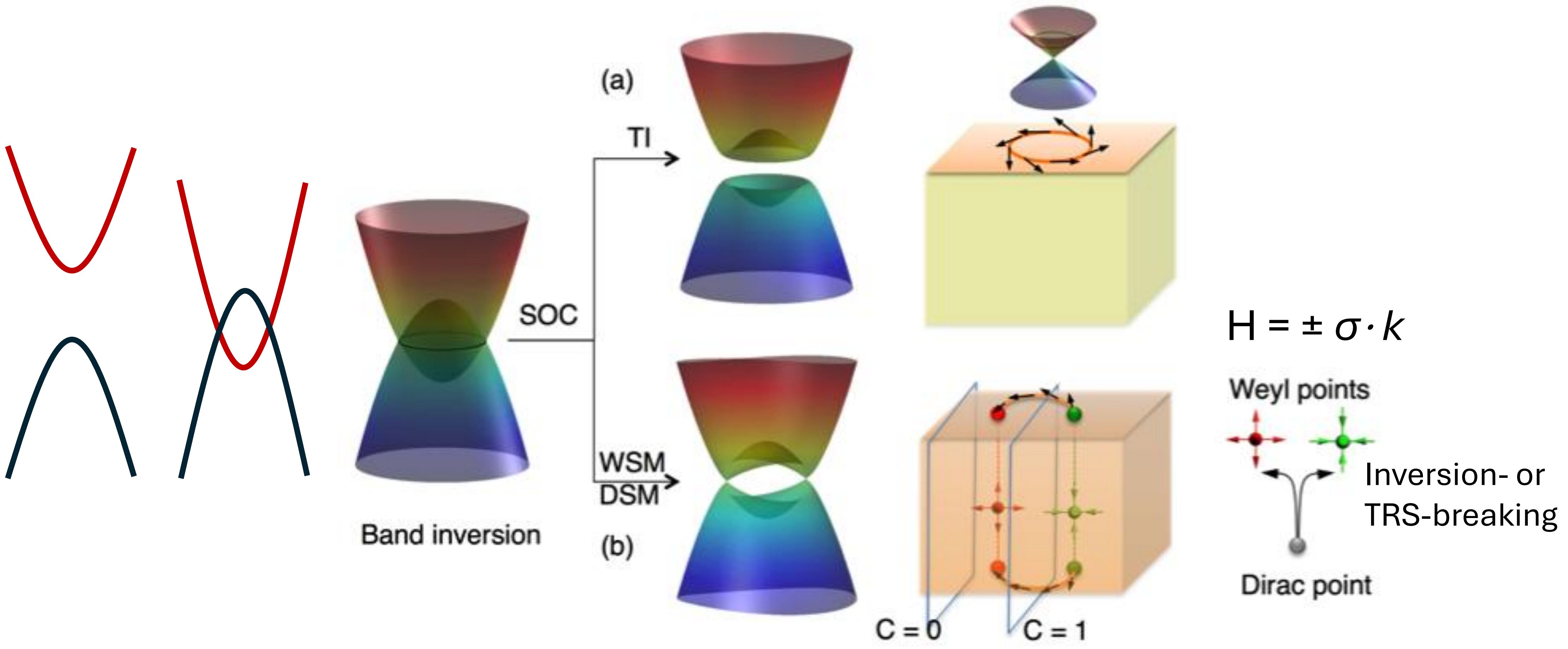


2026 Principles and Applications of Symmetry in Magnetism Summer School
Colorado State University, Jul 16,2026

Topological States of Matter



$H = \pm \sigma \cdot k$

Review: B. Yan and C. Felser, Ann. Rev. Conden. Matter Phys. 8, 337 (2017).

Weyl fermion and Berry curvature

Paul Dirac 1928’

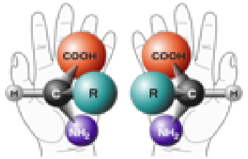
$$\left(\beta mc^2 + c\left(\sum_{n=1}^{3}\alpha_n p_n\right)\right)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

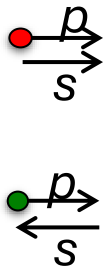
$m = 0$

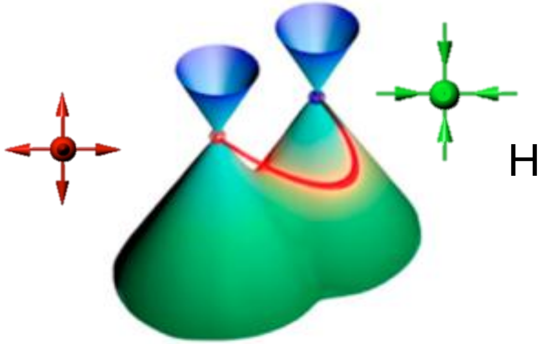
$$i\,\frac{\partial\psi}{\partial t} = \pm\, c\,\vec{p}.\vec{\sigma}\,\psi$$

Hermann Weyl
1929’

H = ± $\boldsymbol{p}\cdot\boldsymbol{\sigma}$



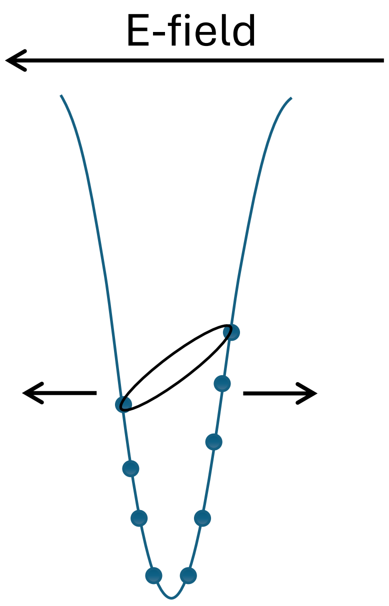




H = ± $\boldsymbol{k}\cdot\boldsymbol{\sigma}$

- Berry curvature $\quad \boldsymbol{\Omega}_{n,xy}(\boldsymbol{k}) = i\langle\nabla_k n| \times |\nabla_k n\rangle = \frac{\widehat{\boldsymbol{k}}}{2k^2}$
- Anomalous velocity $\quad \boldsymbol{v_n} = \frac{d\varepsilon_n}{\hbar d\boldsymbol{k}} - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}_n(k)$

Berry curvature



$$v_n = \frac{1}{\hbar}\nabla_k \varepsilon_n(k)$$

1^{st} order perturbation in $|u_n\rangle$ due to E-field

$$|u_n'\rangle = |u_n\rangle - i\sum_{m\neq n}\frac{|u_m\rangle\langle u_m|\boldsymbol{eEr}|u_n\rangle}{\varepsilon_n-\varepsilon_m}$$

$$v_n' = \langle u_n'|\hat{v}|u_n'\rangle = \langle u_n|\hat{v}|u_n\rangle - i(\langle u_n|\hat{v}|\delta u_n\rangle - c.c.) = \frac{1}{\hbar}\nabla_k\varepsilon_n - \frac{e}{\hbar}\boldsymbol{E}\times\boldsymbol{\Omega}_n$$

$$\Omega_{n,ij} = -2\hbar^2\,\mathrm{Im}\sum_{m\neq n}\frac{\langle u_n|\hat{v}_i|u_m\rangle\langle u_m|\hat{v}_j|u_n\rangle}{(\varepsilon_n-\varepsilon_m)^2}$$

Note:

$$\langle u_m|r_i|u_n\rangle = i\langle u_m|\partial_{k_i}u_n\rangle = i\hbar\frac{\langle u_m|v_i|u_n\rangle}{\varepsilon_n-\varepsilon_m}$$

Berry curvature in transport

$$\boldsymbol{j} = \int_{BZ} (-e) f \cdot \boldsymbol{v}(k) = \int_{BZ} (-e)(f_0 + \delta f)(v_0 + \delta v)$$

$$= \int_{BZ} (-e)\left(f_0 - \frac{\partial f}{\partial k}(-e\boldsymbol{E}\tau)\right)\left(v_0 - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}\right)$$

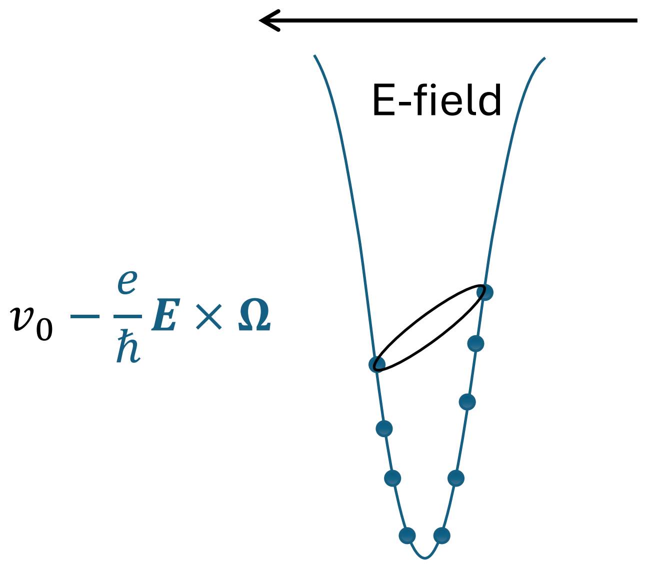
$$= \int_{BZ} -e f_0 v_0 - \frac{e^2}{\hbar}\frac{\partial f}{\partial k} v\tau E + f_0 \frac{e^2}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega} + ..$$

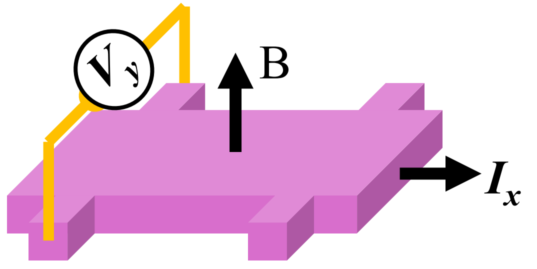
$$\boldsymbol{E} = E_x \hat{\boldsymbol{x}}$$

$$\boldsymbol{j} = \quad 0 + \frac{e^2}{\hbar}\tau \int_{BZ} \frac{\partial f}{\partial \epsilon} v_0^2 \; E_x \hat{\boldsymbol{x}} - \frac{e^2}{\hbar}\int_{BZ} f_0 \Omega_{yx} \, E_x \hat{\boldsymbol{y}}$$

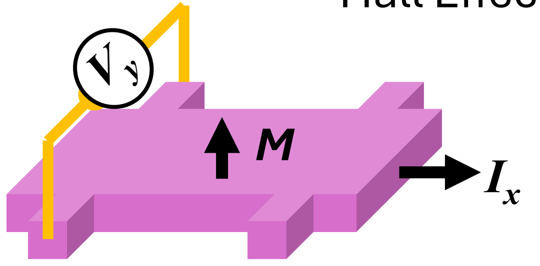
Drude

Anomalous Hall effect (AHE) (without τ)





Hall Effect



AHE

Weyl point and Berry curvature

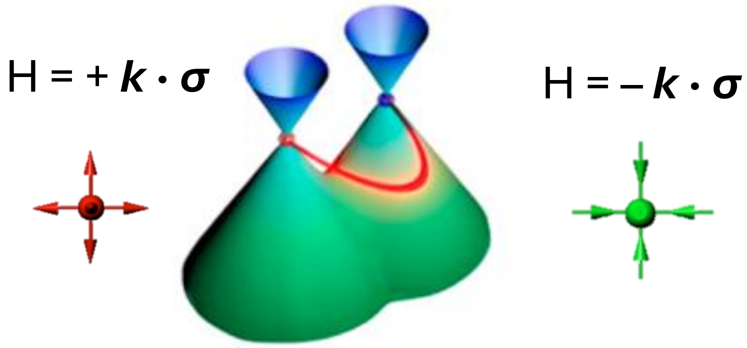
Example: $H = \hbar v k \cdot \sigma$

Choose $(0,0,k_z)$ for example, $H = \hbar k_z \sigma_z$

$\varepsilon_{1,2} = \mp \hbar v\, k_z$; $|u_1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$; $|u_2\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

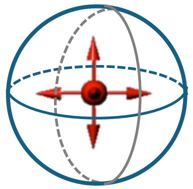
$$\Omega_{n,ij} = -2\hbar^2\,\mathrm{Im}\sum_{m\neq n}\frac{\langle u_n|\hat{v}_i|u_m\rangle\langle u_m|\hat{v}_j|u_n\rangle}{(\varepsilon_n-\varepsilon_m)^2} = \frac{1}{2k_z^2}$$

(Note: $\hat{v}_\mathrm{i} = \frac{dH}{\hbar dk_\mathrm{i}} = v\sigma_\mathrm{i}$)



Berry curvature (magnetic field in k-space)
Berry phase (flux)

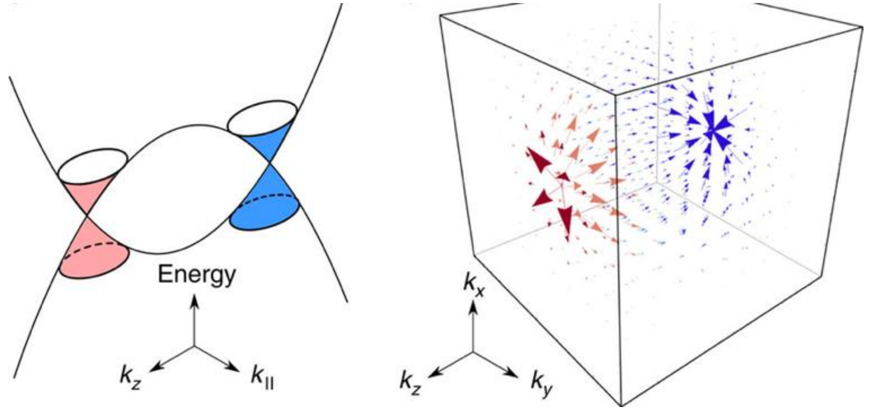
$\boldsymbol{\Omega_1}(\boldsymbol{k}) = \frac{\widehat{\boldsymbol{k}}}{2k^2}$



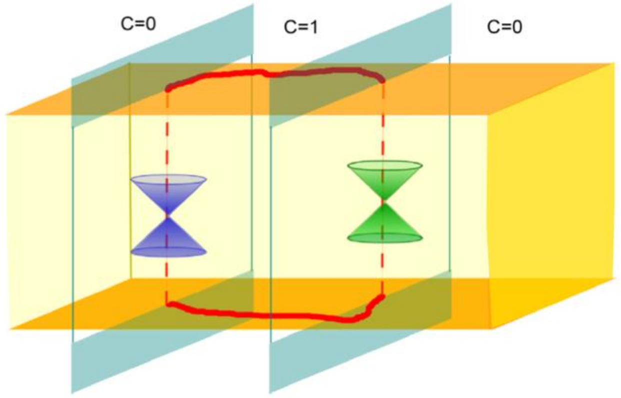
$\frac{\mathbf{1}}{\mathbf{2\pi}}\oiint \boldsymbol{d^2k}\,\boldsymbol{\Omega_1}(\boldsymbol{k}) = \frac{1}{2\pi}4\pi k^2\frac{1}{2k^2} = 1$

Quantized flux
Monopole charge

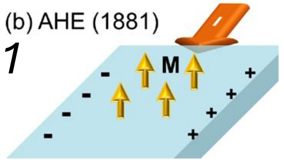
AHE, Quantum AHE and Fermi arcs



Fermi arcs



C=1



Quantum AHE

$$\sigma_{xy}^{2D} = -\frac{e^2}{h}\int f_0\,\Omega(k)dk = \frac{e^2}{h}C$$

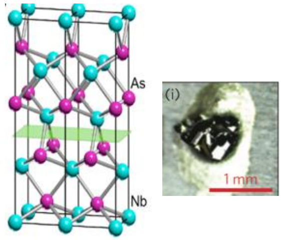
Haldane 1988

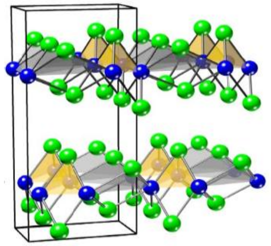
AHE in Weyl

$$\sigma_{xy}^{3D} = \frac{e^2}{h}\frac{k_0}{2\pi}$$

Weyl semimetal materials

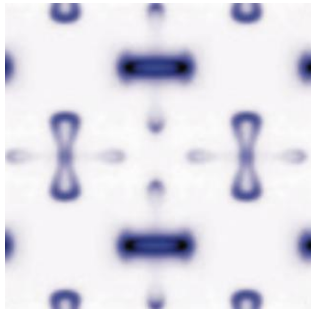
Nonmagnetic (Inversion-breaking)

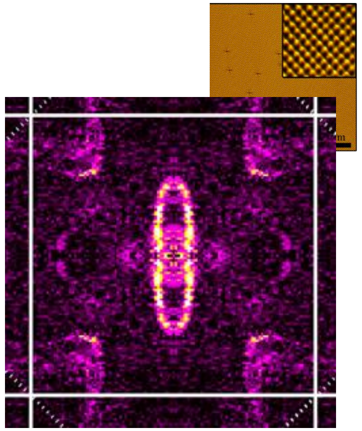




TaAs, TaP, NbAs, NbP

WTe_2, $MoTe_2$

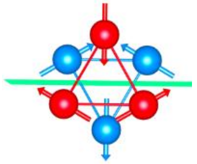


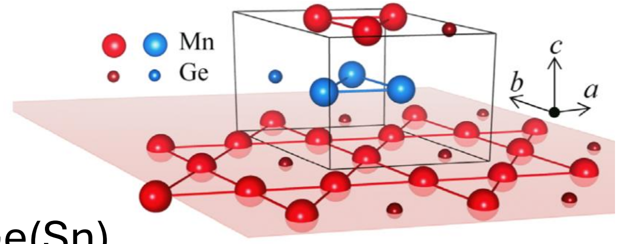


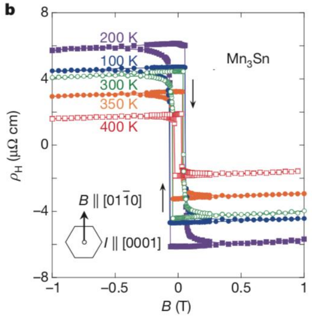
S.-Y. Xu et al., Science **349**, 613 (2015).
B. Q. Lv et al., Phys. Rev. X **5**, 031013 (2015).
L.X. Yang et al. Nature Physics **11**, 728 (2015).
R. Batabyal et al. Science Adv. 2, e1600709 (2016)

B. A. Bernevig et al. Nature **527**, 495 (2015).
B. Yan et al. PRB **92**, 161107 (2015).

Magnetic (Time-reversal breaking)

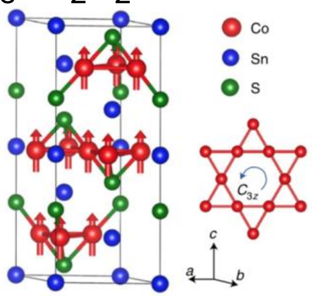


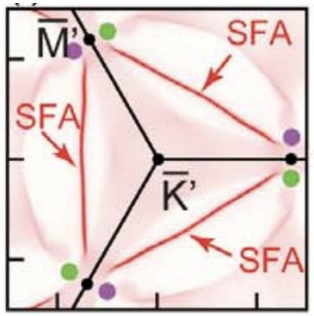


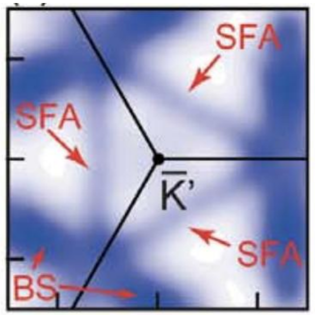


$Mn_3Ge(Sn)$

$Co_3Sn_2S_2$



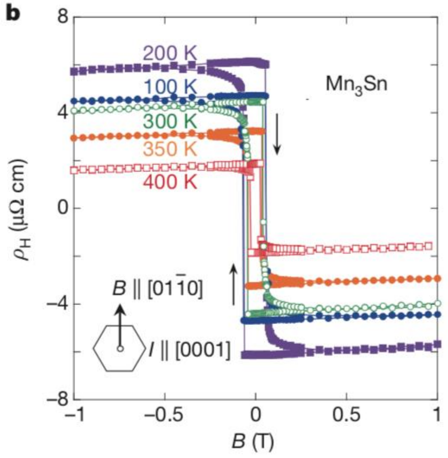


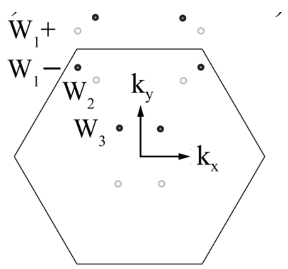


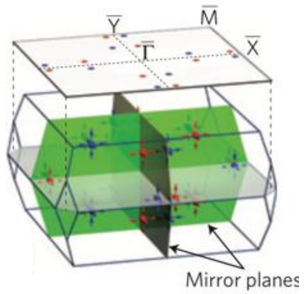
Yang *et al* *New J. Phys.* **19** 015008 (2017)
Nakatsuji et al. Nature 527, 212 (2015)
Nayak et al. Sci. Adv. 2, e1501870 (2016)

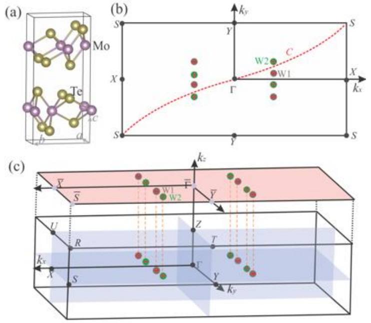
Liu et al. Nature Physics 14, 1125 (2018)
Morali et al. Science 365, 1286 (2019)
Liu et al., Science 365, 1282 (2019)

Anomalous Hall Effect









Mn$_3$Sn(Ge), Co$_3$Sn$_2$S$_2$

TaAs

MoTe$_2$/WTe$_2$

$$\sigma_{xy}^{3D} = -\frac{e^2}{h}\int f_0\, \Omega(k) dk$$

AHE

$\sigma_{xy}^{3D} = 0$ because $\Omega(k) = -\Omega(-k)$

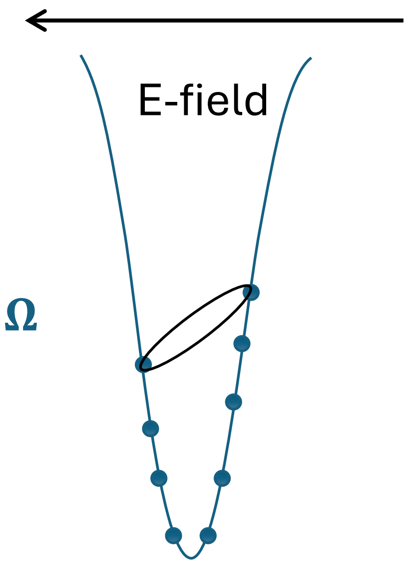
No AHE

In the presence of TRS, will monopoles lead to some exotic transport phenomena?

Berry curvature in transport

$$\boldsymbol{j} = \int_{BZ} (-e) f \cdot \boldsymbol{v}(k) = \int_{BZ} (-e)(f_0 + \delta f)(v_0 + \delta v)$$

$$v_0 - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}$$



$$= \int_{BZ} (-e)\left(f_0 - \frac{\partial f}{\partial k}(-e\boldsymbol{E}\tau)\right)\left(v_0 - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}\right)$$

$$= \int_{BZ} -ef_0 v_0 - \frac{e^2}{\hbar}\frac{\partial f}{\partial k} v\tau E + f_0 \frac{e^2}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega} - \frac{e^3}{\hbar^2}\frac{\partial f}{\partial k} E(\boldsymbol{E} \times \boldsymbol{\Omega})\tau$$

$$\boldsymbol{E} = E_x \hat{\boldsymbol{x}}$$

$$\boldsymbol{j} = \quad 0 + \frac{e^2}{\hbar}\tau \int_{BZ} \frac{\partial f}{\partial \epsilon} v_0^2 \; E_x \hat{\boldsymbol{x}} - \frac{e^2}{\hbar} \int_{BZ} f_0 \Omega_{yx} \, E_x \hat{\boldsymbol{y}} - \frac{e^3}{\hbar^2}\tau \int_{BZ} \frac{\partial f}{\partial k_x} \Omega_{yx} \, E_x^2 \hat{\boldsymbol{y}}$$

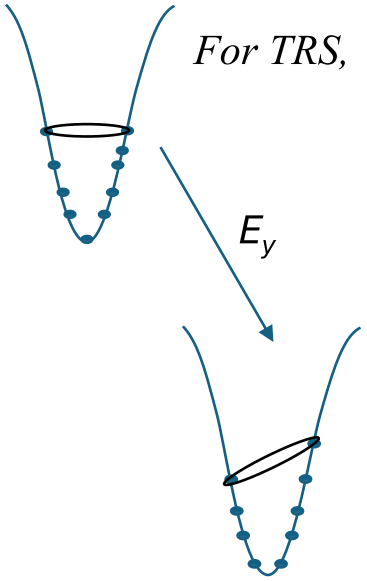
Drude

Anomalous Hall effect (AHE) (without τ)

Nonlinear AHE

WSM – Transport phenomena

Anomalous velocity $$\boldsymbol{v}_n(\boldsymbol{k}) = \frac{\partial \varepsilon_n(\boldsymbol{k})}{\hbar \partial \boldsymbol{k}} - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}_n(\boldsymbol{k})$$



Non-equilibrium Fermi surface

For TRS, $\Omega_n(k) = -\Omega_n(-k)$

$$\sigma_{xy} = \Sigma_n \int \Omega_n(k) f_0(E_k) dk = 0$$

$$\sigma'_{xyy} = \int \Omega_n(k)\left(f_0 + \frac{\partial f_0}{\partial k}\right) dk \neq 0$$

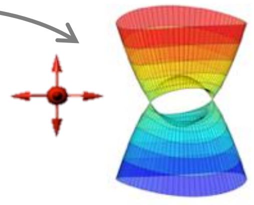
$$= \int \Omega_n(k)\left(\frac{\partial f_0}{\partial k}\right) dk = \int f_0 \left(\frac{\partial \Omega_n(k)}{\partial k}\right) dk$$

Linear Response

$$j_x = \sigma_{xy} E_y$$

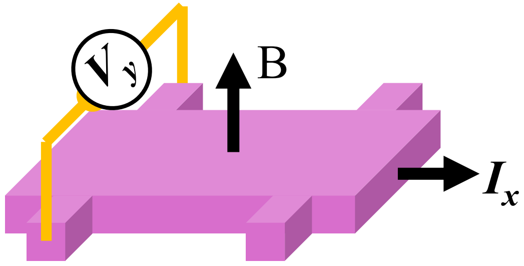
Nonlinear Response

$$j_x = \sigma'_{xyy}\, E_y^2$$



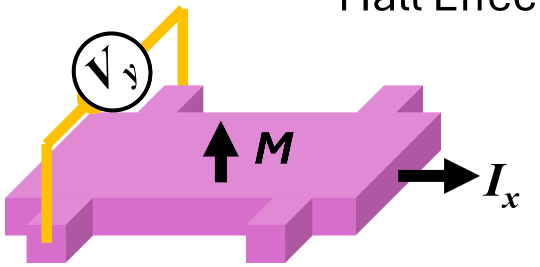
Nonlinear Anomalous Hall effect

$$\boldsymbol{j} = \sigma_{xx} E_x \hat{\boldsymbol{x}} + \sigma_{yx} E_x \hat{\boldsymbol{y}} + \sigma_{xx}^{y} E_x^2 \hat{\boldsymbol{y}}$$

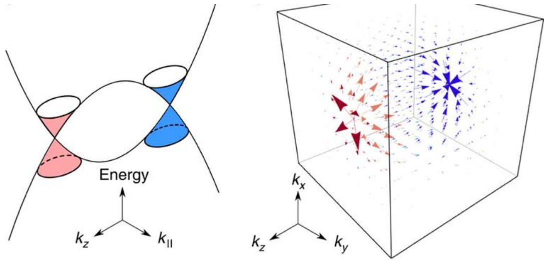


Hall Effect

$$\sigma_{xx}^{y} = -\frac{e^3}{\hbar}\tau \int \frac{df_0}{dk} \boldsymbol{\Omega} dk = -\frac{e^3}{\hbar}\tau \int f_0 \frac{\partial \boldsymbol{\Omega}}{\partial k} dk$$



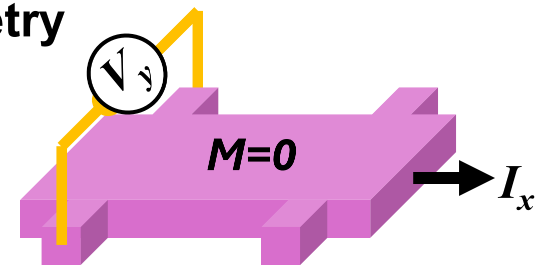
AHE



NLAHE survives with time-reversal symmetry

$$\Omega(\mathrm{k}) = -\Omega(-\mathrm{k})$$

$$\frac{\partial \Omega(\mathrm{k})}{\partial k} = +\frac{\partial \Omega(-\mathrm{k})}{\partial(-k)}$$



Nonlinear AHE

I. Sodemann and L. Fu, PRL 115, 216806 (2015)

A Weyl toy model

A single Weyl cone

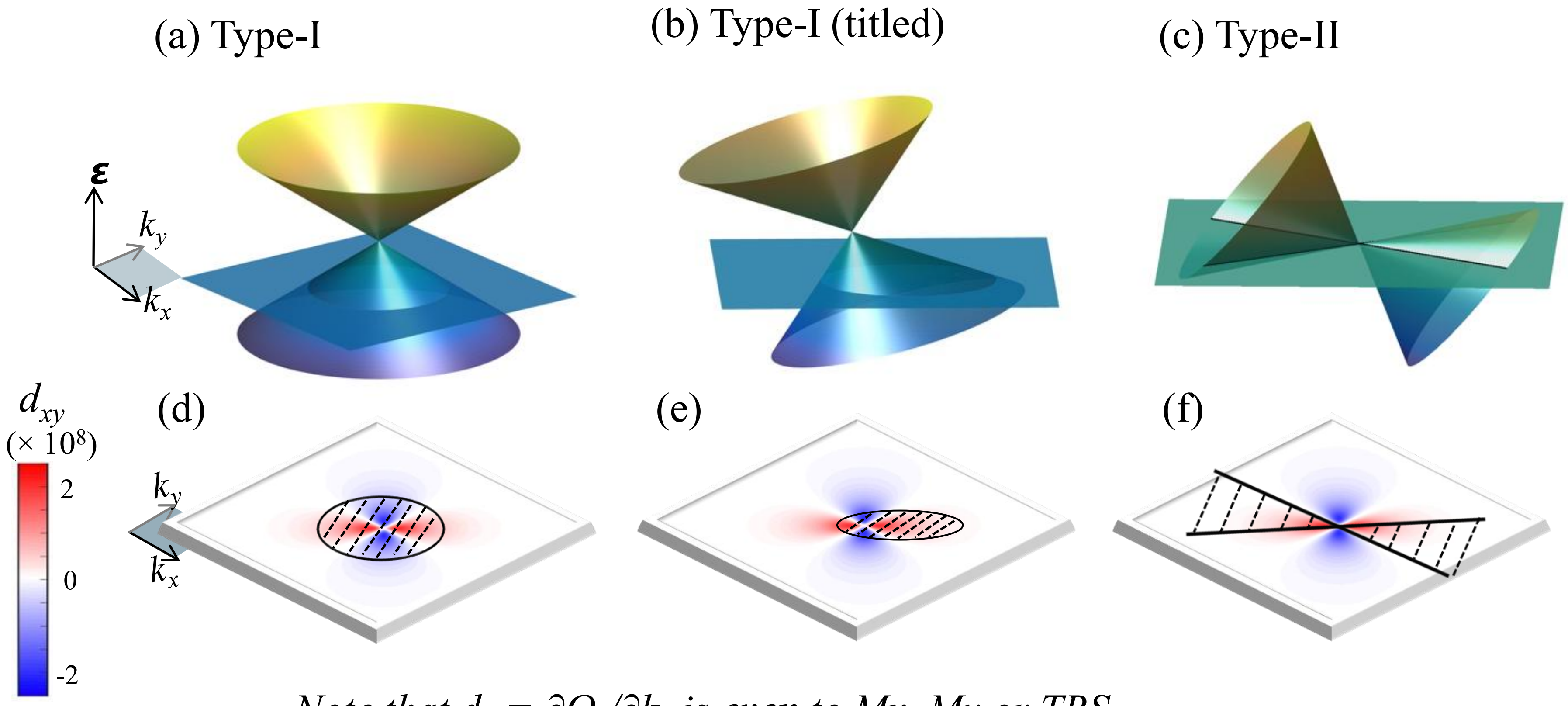
$$H_{Weyl}(\mathbf{q}) = \hbar v_t q_t \sigma_0 + \hbar v_F\, \mathbf{q}\cdot\sigma,$$

Berry curvature

$$\Omega(\mathbf{q}) = \frac{\mathbf{q}}{2q^3}$$

Berry curvature dipole

$$d_{xy} = \frac{\partial \Omega_y}{\partial q_x} = \frac{3q_x q_y}{2q^5}$$



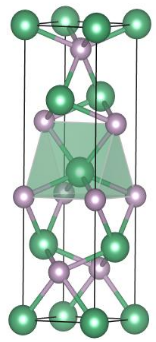
*Note that $d_{xy} = \partial\Omega_y/\partial k_x$ is **even** to Mx, My or TRS.*
So a pair of Weyl points contribute the same D_{xy}.

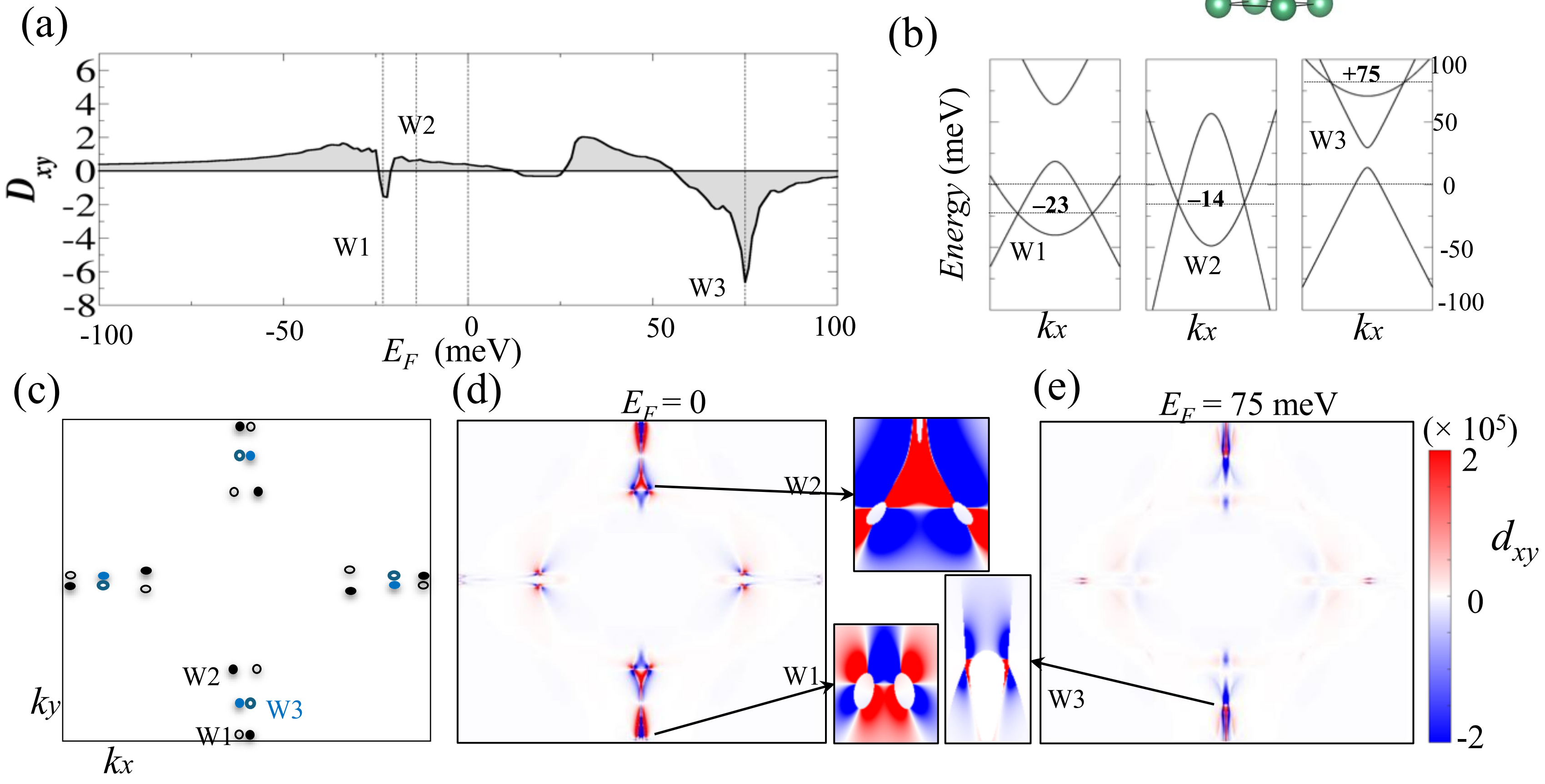
Berry curvature dipole for TaAs

- Point group C4v: M_x, M_y, C_4

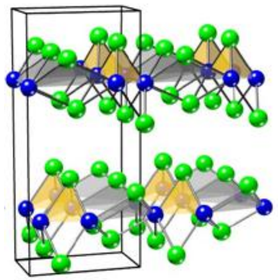
E.g. Mx: $k_x \rightarrow -k_x$, $\Omega_x \rightarrow \Omega_x$ *so* $d_{xx} \rightarrow -d_{xx}$ *then* $D_{xx} = 0$

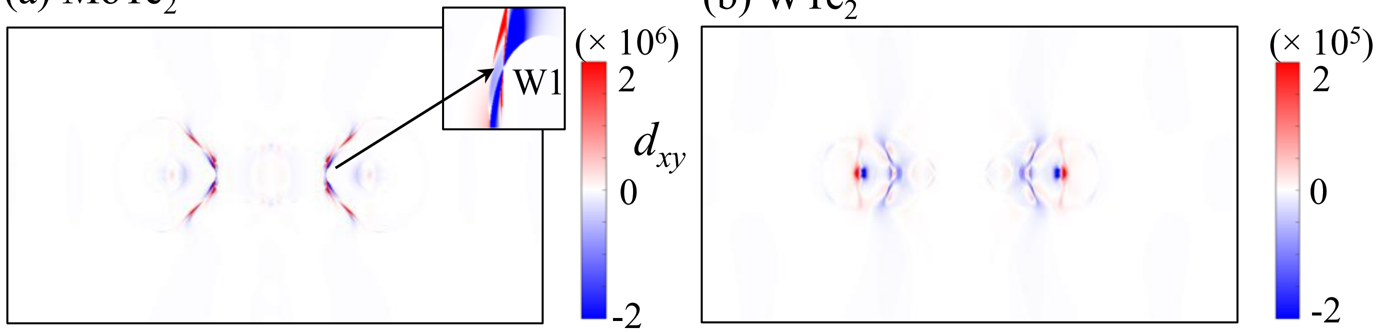
Finally we have ***only one*** *independent element* $D_{xy} = -D_{yx}$





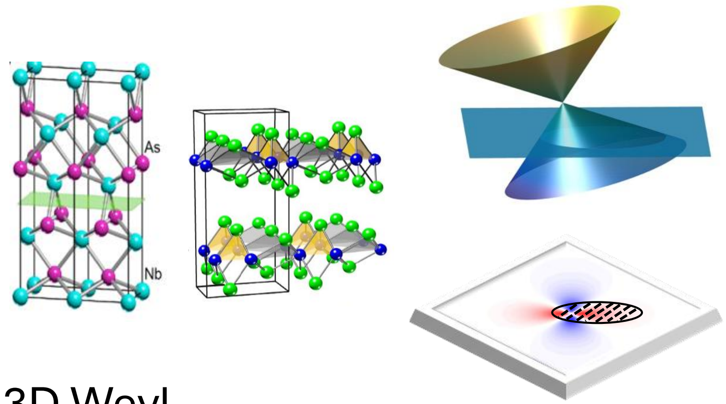
Berry curvature dipole for $MoTe_2$ and WTe_2





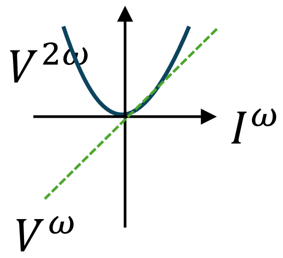
Nonlinear Anomalous Hall effect

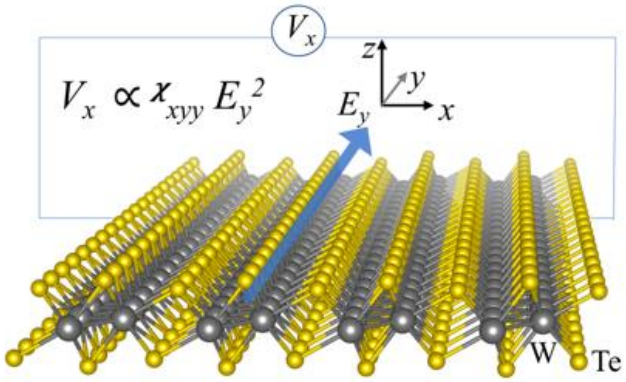
Ab initio $\boldsymbol{u_n}(\mathbf{k})$	$\rightarrow$	$\boldsymbol{\Omega}(\boldsymbol{k}) = i\langle\nabla_k n| \times |\nabla_k n\rangle$
Next step	$\rightarrow$	$\dfrac{\boldsymbol{\partial\Omega(k)}}{\boldsymbol{\partial k}}$



3D Weyl

$\sigma^{(2)} \sim 0.1 - 1\%\ \sigma^{(1)}$





2D Weyl layers WTe_2 and $MoTe_2$, **gate tuning E_F**

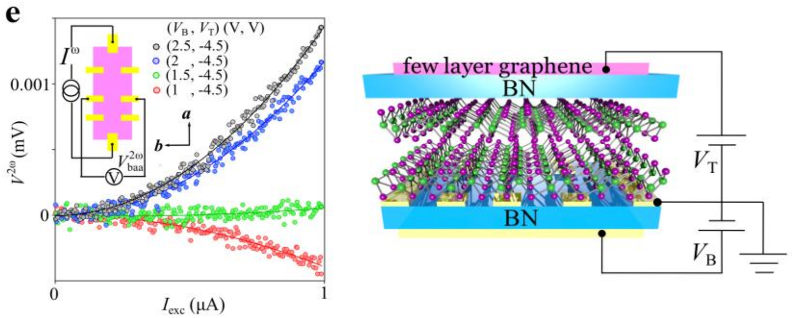
Y. Zhang, B. Yan et al. PRB 97, 041101(R) (2017).

Y. Zhang, B. Yan et al. 2D Mater. 5 044001 (2018).

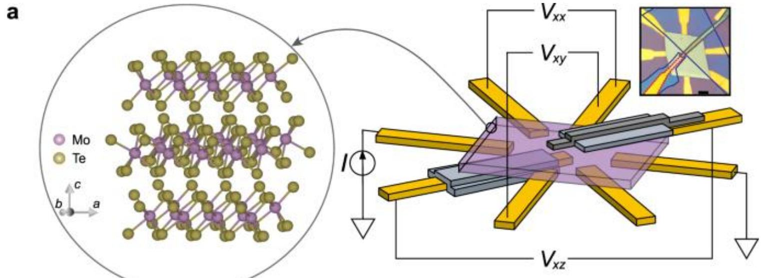
Very recent realization in WTe_2 and $MoTe_2$

2D

3D



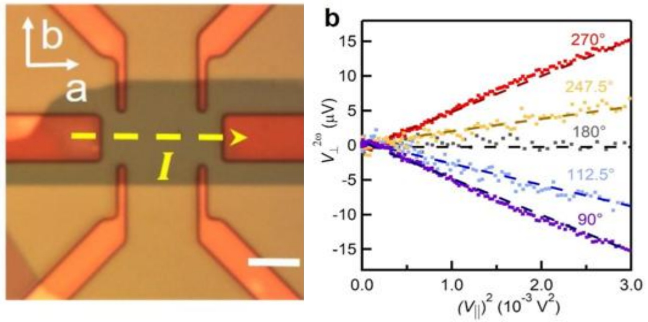
Q. Ma, P. Jarillo-Herrero, et al, *Nature* **565**, 337–342 (2019)



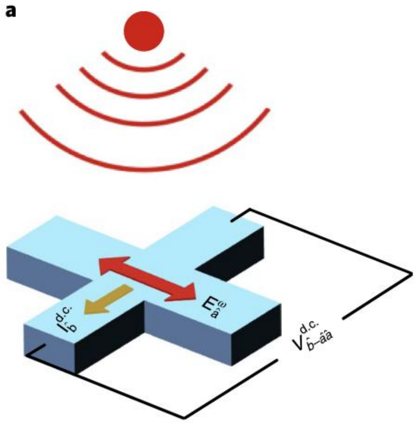
Tsen et al. Nature Comm. 12, 2049 (2021) 3D $MoTe_2$

Z. Mao, B. Yan et al., Nature Comm. (2022) Room temperature, BaMnSb2

$V_\perp^{2\omega}$ *is* 0.1% *of* $V_{||}$



Mak and Shan, et al, *Nature Materials* **18**, 324–328 (2019)



Wireless radiofrequency (RF) rectification with zero external bias, TaIrTe4

H. Yang et al. Nautre Nano 2021

Nonlinear optics

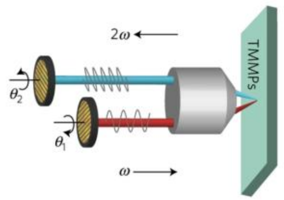
Shining light $E = E_0\cos(\omega t)$,
Polarization $P = \chi^{(1)}E + \chi^{(2)}E^2 + \ldots$
DC photocurrent $J_{DC} = dP/dt = \cancel{\sigma^{(1)}E} + \sigma^{(2)}E^2 + \ldots.$

Studied in polar insulators such as $BaTiO_3$
Theory: shift current, injection current ..

Bulk Photovoltaic Effect (BPVE)

Lighting up the Weyl semimetal?

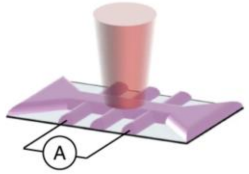
Second Harmonic Generation

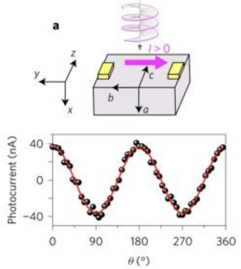


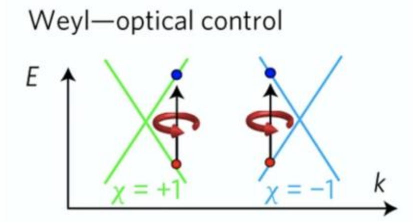
Material	$\lvert d_{ij} \rvert$	$\lvert d \rvert$ (pm V^{-1})	Fundamental wavelength (nm)
TaAs	d_{33}	3600	800
$BaTiO_3$	d_{33}	15	900
$BiFeO_3$	d_{33}	15–19	1550, 800
$LiNbO_3$	d_{33}	26	852

L. Wu, J. Orenstein et al. Nature Phys 2017

Photocurrent



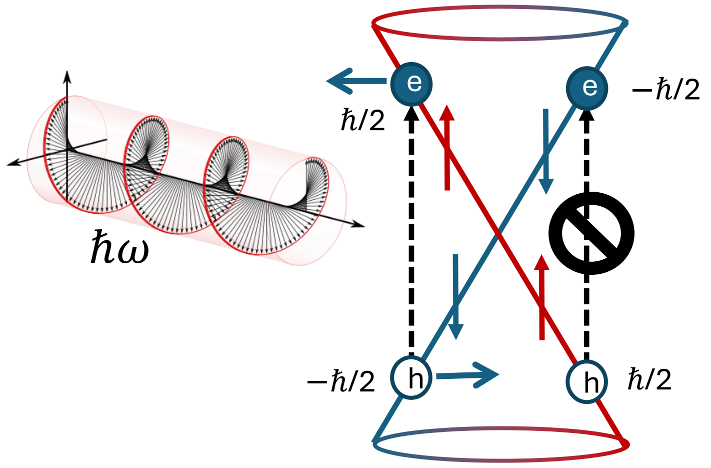




G. Osterhoudt, K. Burch et al. Nature Mater 2017
Q. Ma, N. Gedik et al. Nature Phys 2017

Topological nature of optical responses with TRS

Injection current

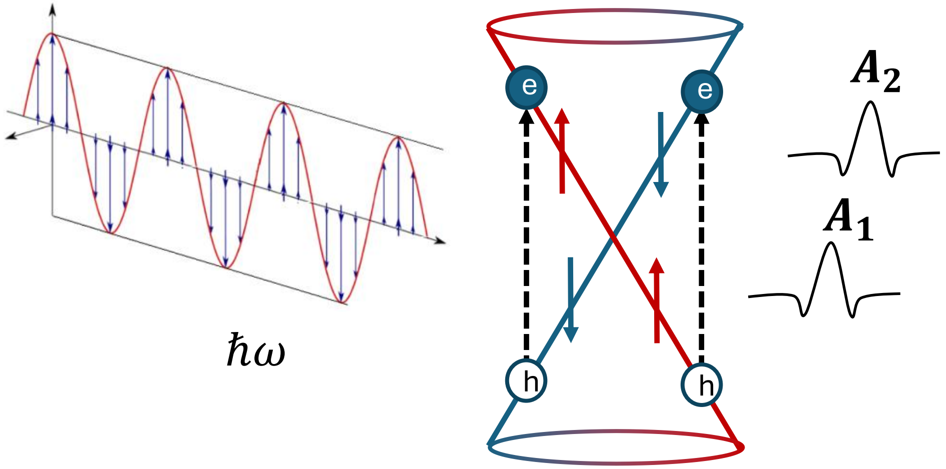


Weyl cone in **circularly-polarized** light

$$\frac{dJ_c}{dt} \propto \boldsymbol{\Omega}^{ab}_{nm}(v^c_n - v^c_m) f_{nm} E^2$$

J. Sipe PRB 2000

Shift current

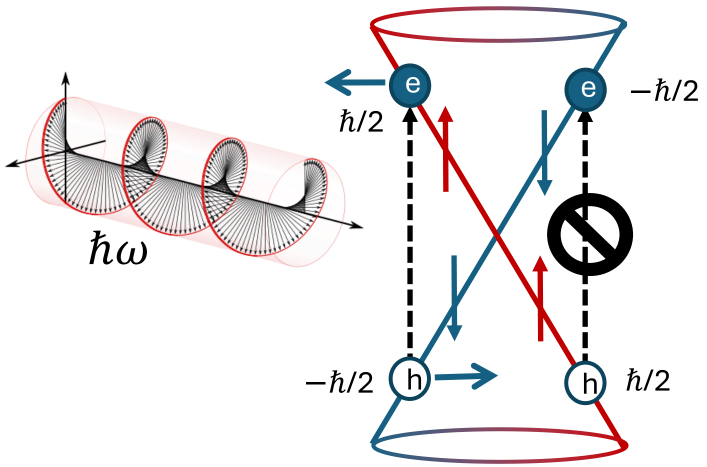


$$J \propto (A_1 - A_2 - \partial_k \phi_{12}) E^2$$

R. von Baltz and W. Kraut 1981

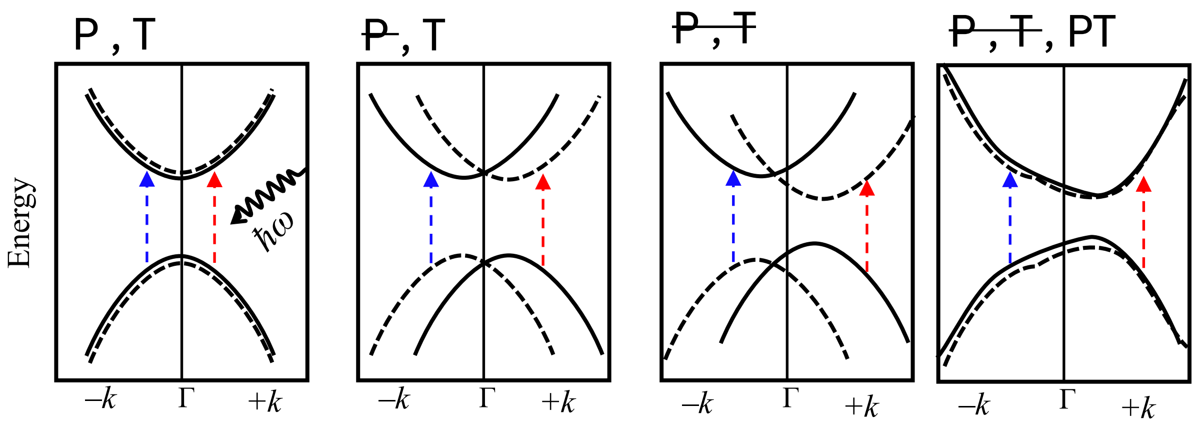
Design a new injection current

Excitation asymmetry

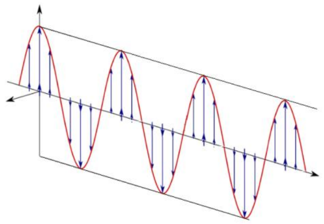


Weyl cone in **circularly-polarized** light

Band structure asymmetry

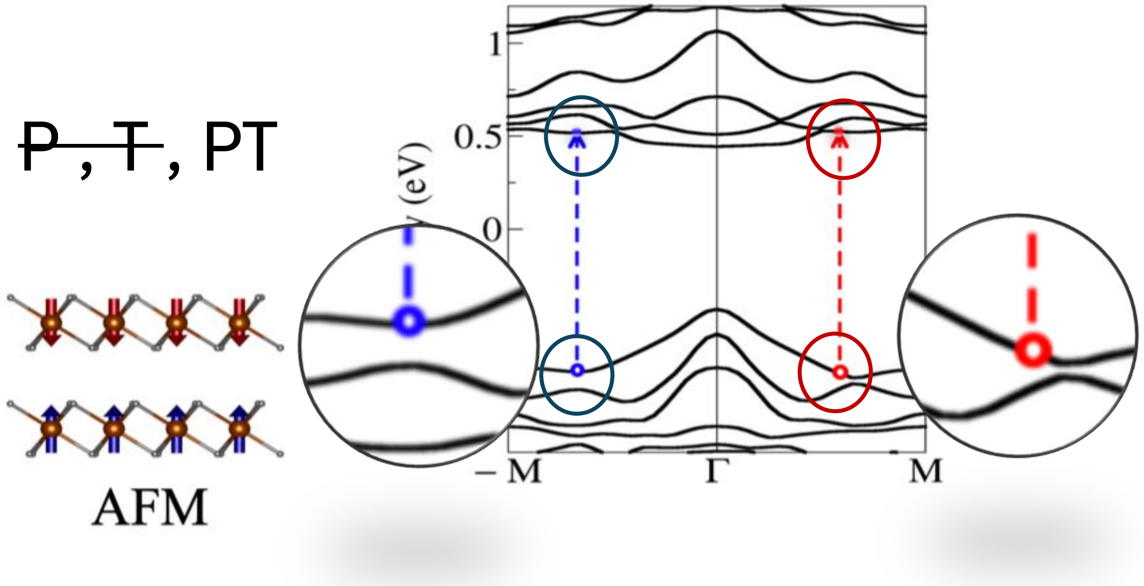


Linearly-polarized / nonpolarized light?

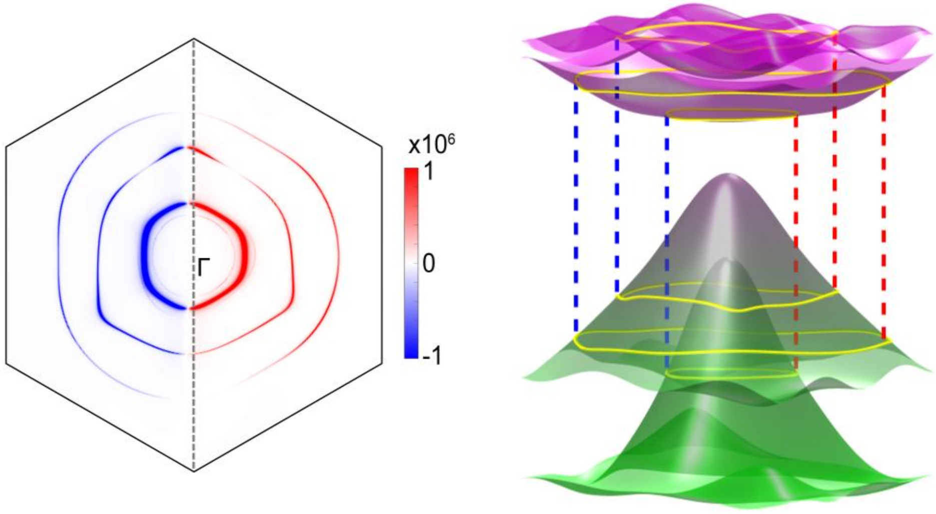


Magnetic Injection Current ($\hbar\omega > Eg$)

~~P~~, ~~T~~, PT



Asymmetric Fermi surface



$$\sigma_{aa;c} \propto \boldsymbol{g_{nm}^{aa}}(v_n^c - v_m^c)\tau$$
$$\boldsymbol{g_{nm}^{aa}} = |\boldsymbol{r_{nm}^{a}}|^2$$

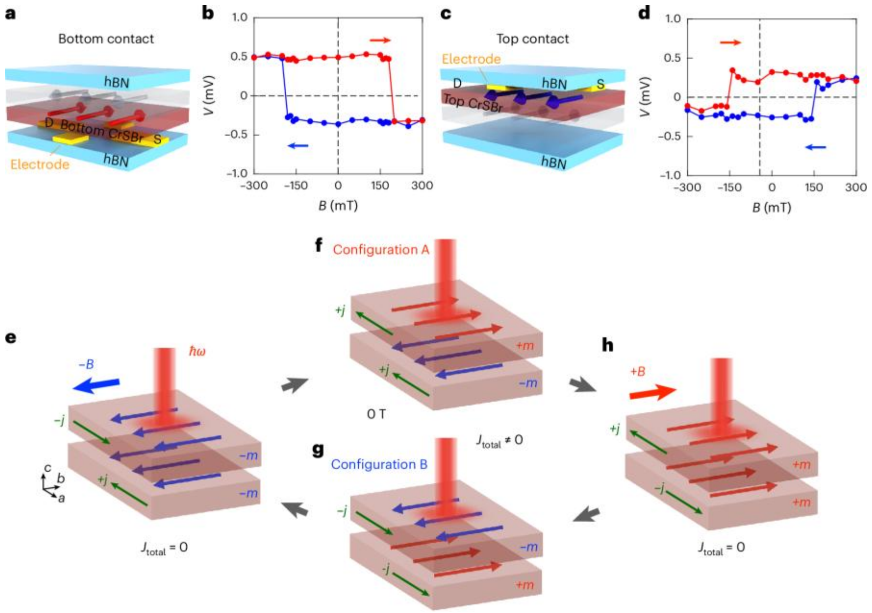
Magnetic quantum materials:
CrI_3 bilayer, Cr_2O_3, $MnPSe_3$, CuMnAs, Mn_2Au, doped-magnetic TIs, thin film $MnBi_2Te_4$

Zhang et al. Nature Comm. 10, 3783 (2019)
T. Holder, D. Kaplan, B. Yan PRR 2020

Recent discovery of magnetic photocurrent

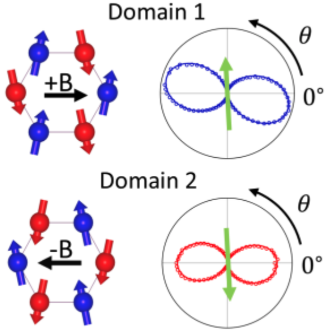
CrSBr AFM

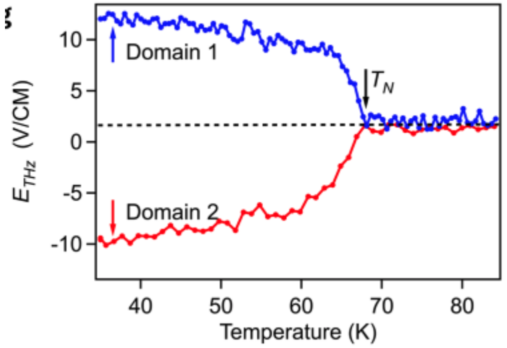
Dong et al, Nature Mater. (2026)

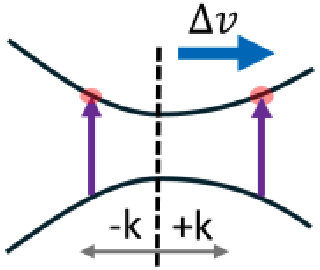


MnPSe3 AFM

Tian et al. arXiv:2605.22518







Summary

	Transport	Optics
Berry curvature	Anomalous Hall effect (τ-independent)	CPGE, injection current (τ-linear)
Berry curvature dipole	Nonlinear AHE (τ-linear)	
Berry connection		LPGE, shift current (τ-independent)
Quantum metric	??	Magnetic injection current (τ-linear)