Topological and Multipolar Magnets and Spintronics

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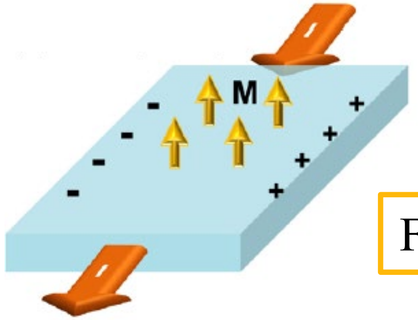
Plan

- Multipole Physics on Correlated Electron Systems
- Topological States in Magnetic Systems
- Physics of Antiferromagnetic Weyl Semimetals
- Physics of Multipolar Kondo Lattice Systems

Order parameters characterizing AHE

● Ferromagnets

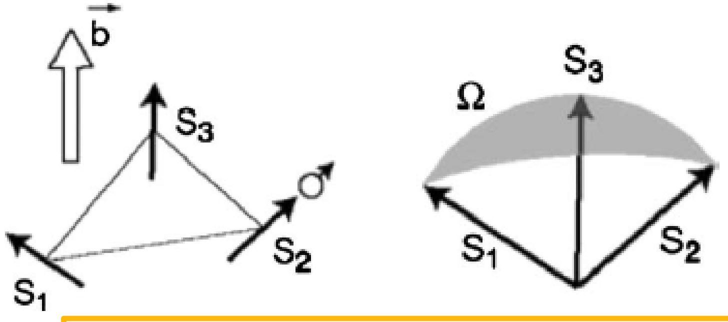


Ferromagnets exhibit AHE in the presence of S-O coupling.

Karplus and Luttinger, Phys. Rev. **95**, 1154 (1954)

Figure from C.-Z. Chang and M. Li. (2016)

● Non-coplanar ferromagnets

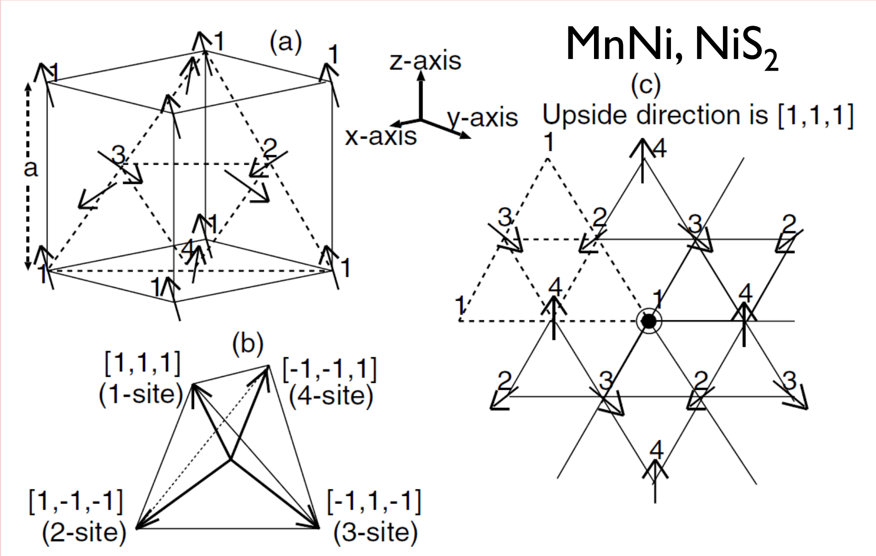


$$\chi_{ijk} = \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

Finite scalar spin chirality induces the AHE.

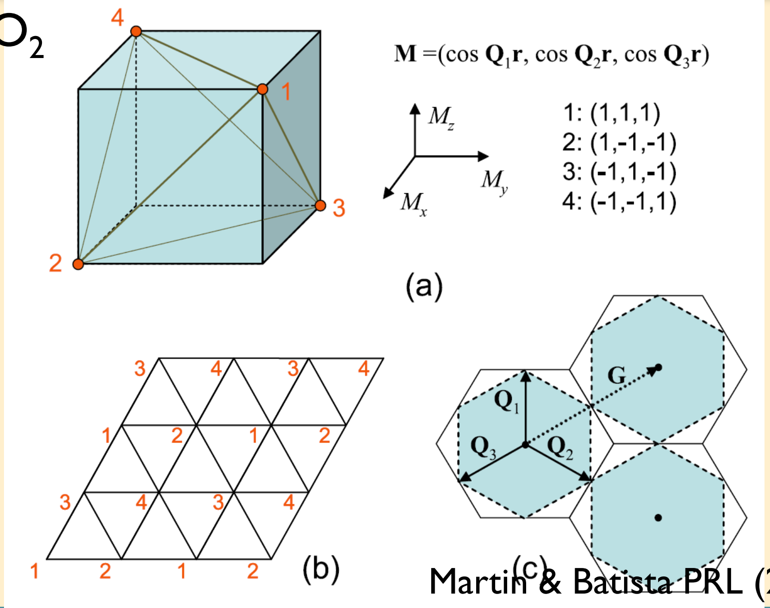
Shindou and Nagaosa, Phys. Rev. Lett. 87, 116801 (2001).

Theory: AHE in AFM

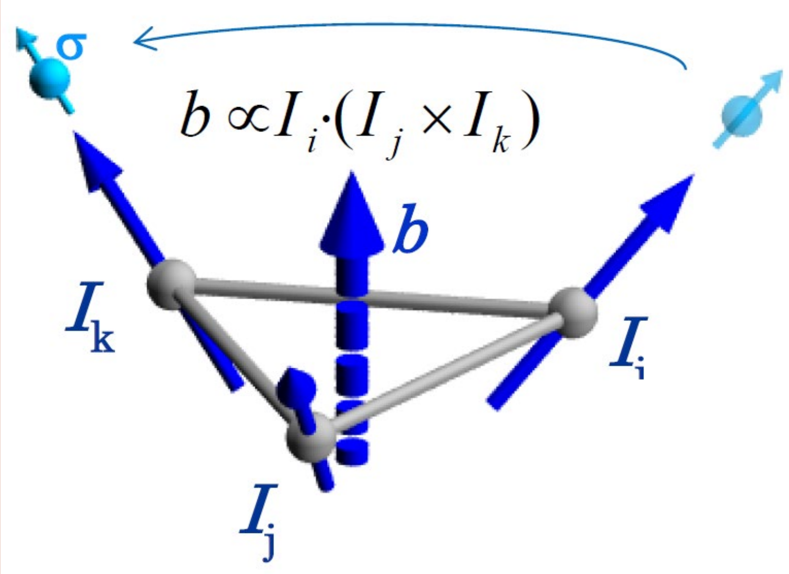


Shindou & Nagaosa PRL (2001).

$NaCoO_2$

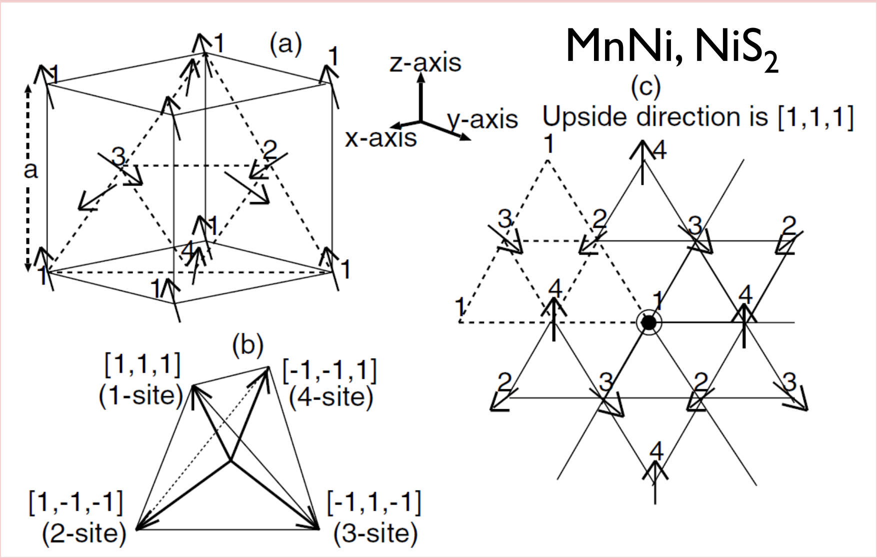


Martin & Batista PRL (2008).



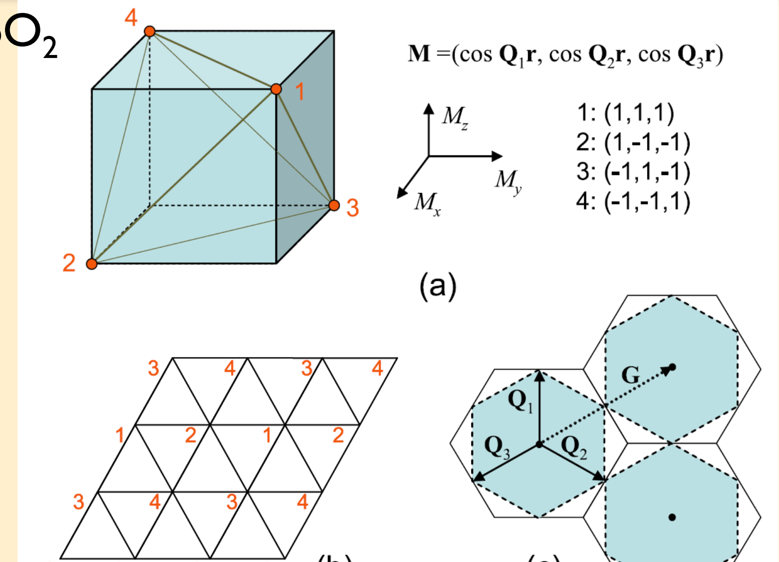
Spin Chirality Mechanism

Theory: AHE in AFM

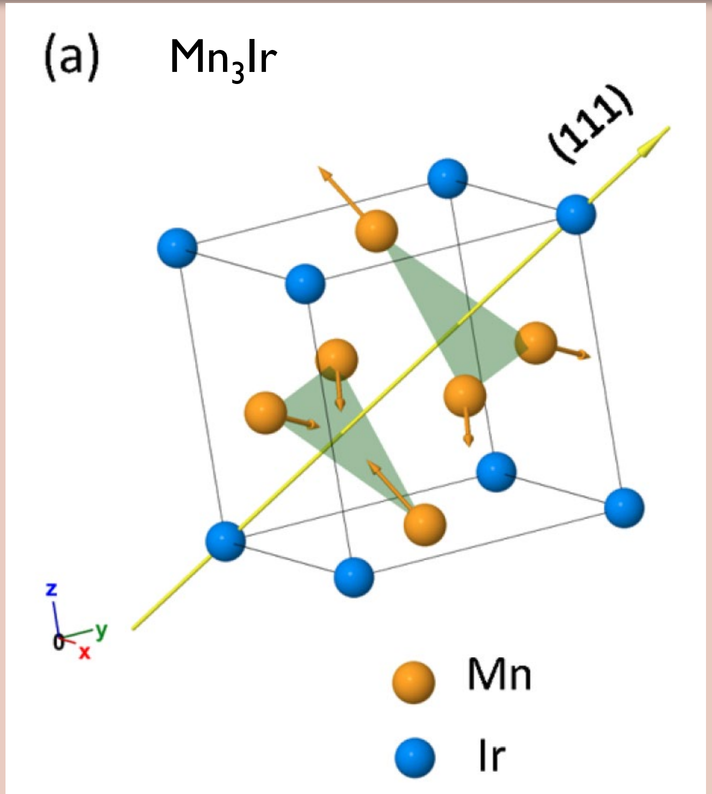


Shindou & Nagaosa PRL (2001).

$NaCoO_2$



Martin & Batista PRL (2008).



Chen, Niu, MacDonald, PRL (2014).

What is the order parameter behind AHE?

Magnetic Multipoles vs. Cluster Multipoles

Suzuki, Arita et al., PRB 094406(2017).

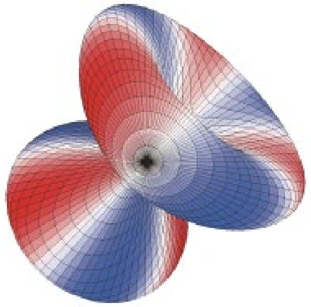
Magnetic multipoles

$$M_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}} \int d\boldsymbol{r} \nabla_i (r^\ell Y_{\ell m}(\hat{\boldsymbol{r}})^*) \cdot \boldsymbol{m}(\boldsymbol{r})$$

Magnetization density around an atom

Magnetic dipole

Magnetic octupole



Characterize the **magnetization density** around **an atom**

Symmetry properties

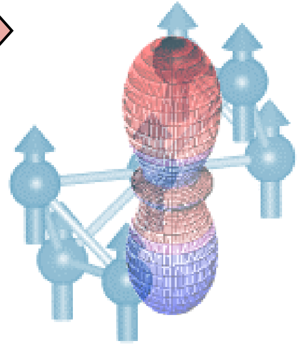
Cluster multipoles (CMP)

$$M_{\ell m} \equiv \sqrt{\frac{4\pi}{2\ell+1}} \sum_{i=1}^{N} \nabla_i (R_i^\ell Y_{\ell m}(\hat{\boldsymbol{R}}_i)^*) \cdot \boldsymbol{m}_i$$

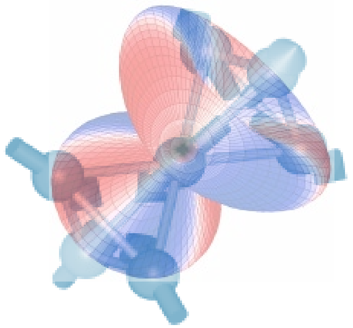
magnetic moment of the *i*-th atom

Suzuki, RA, et al., PRB 95 094406(2017)

Cluster dipole
(Ferromagnet)



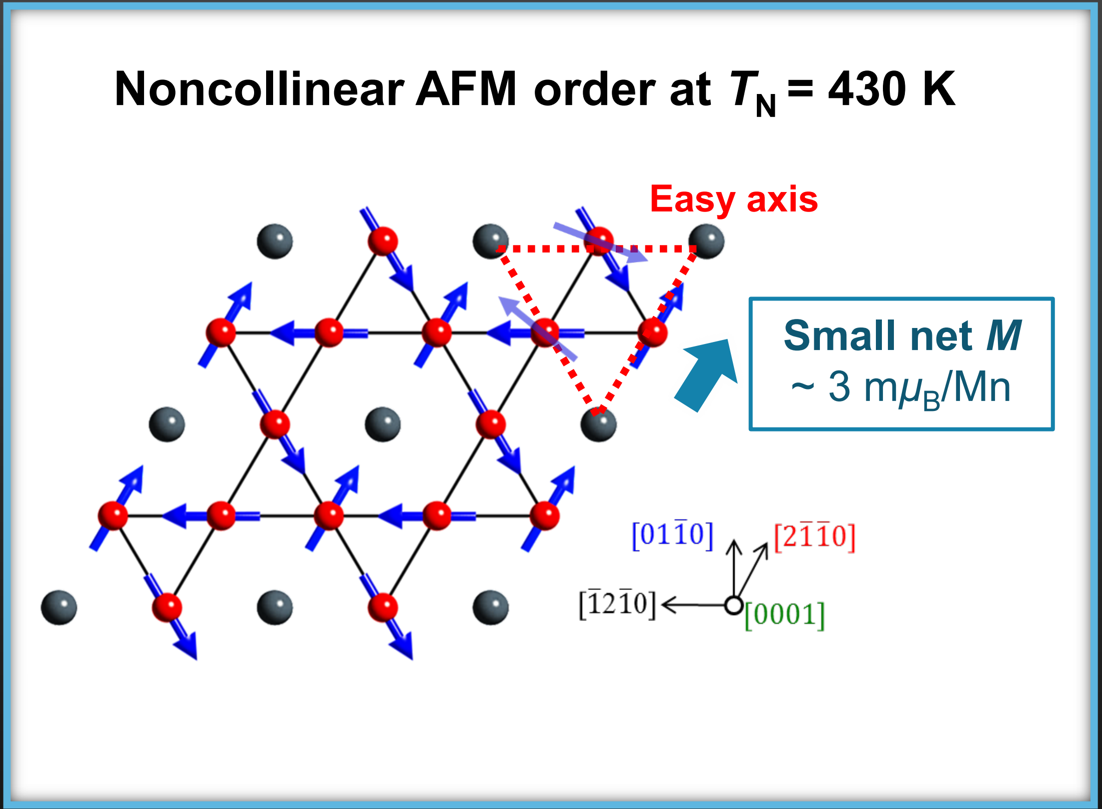
Cluster octupole
(Antiferromagnet)



Characterize the **magnetic configuration** for **a cluster of atoms**

CMP: A new basis for classifying antiferromagnetic structures

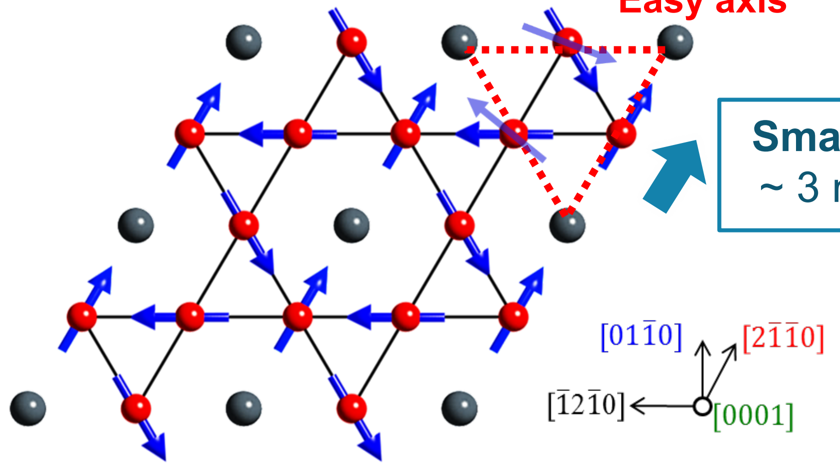
Large room-temperature AHE in AFM Mn_3Sn



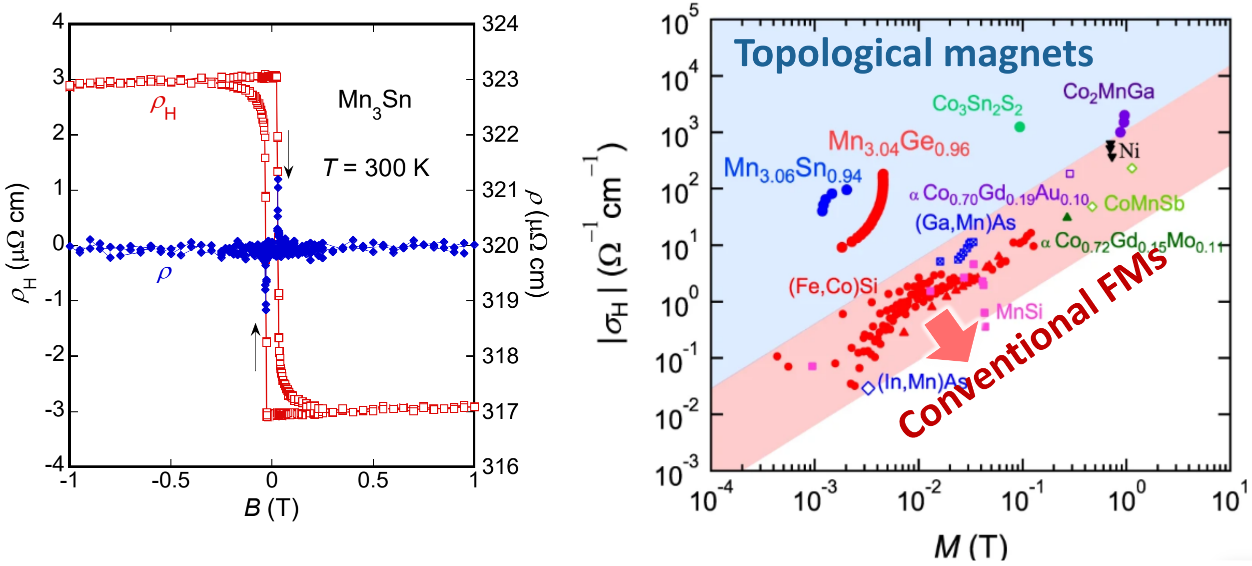
$\rho_H = R_0 B + R_S \mu_0 M \sim 0.01\ \mu\Omega\text{cm}$

Large room-temperature AHE in AFM Mn_3Sn

Noncollinear AFM order at T_N = 430 K



Large AHE well beyond the linear-in-*M* relation for conventional FMs

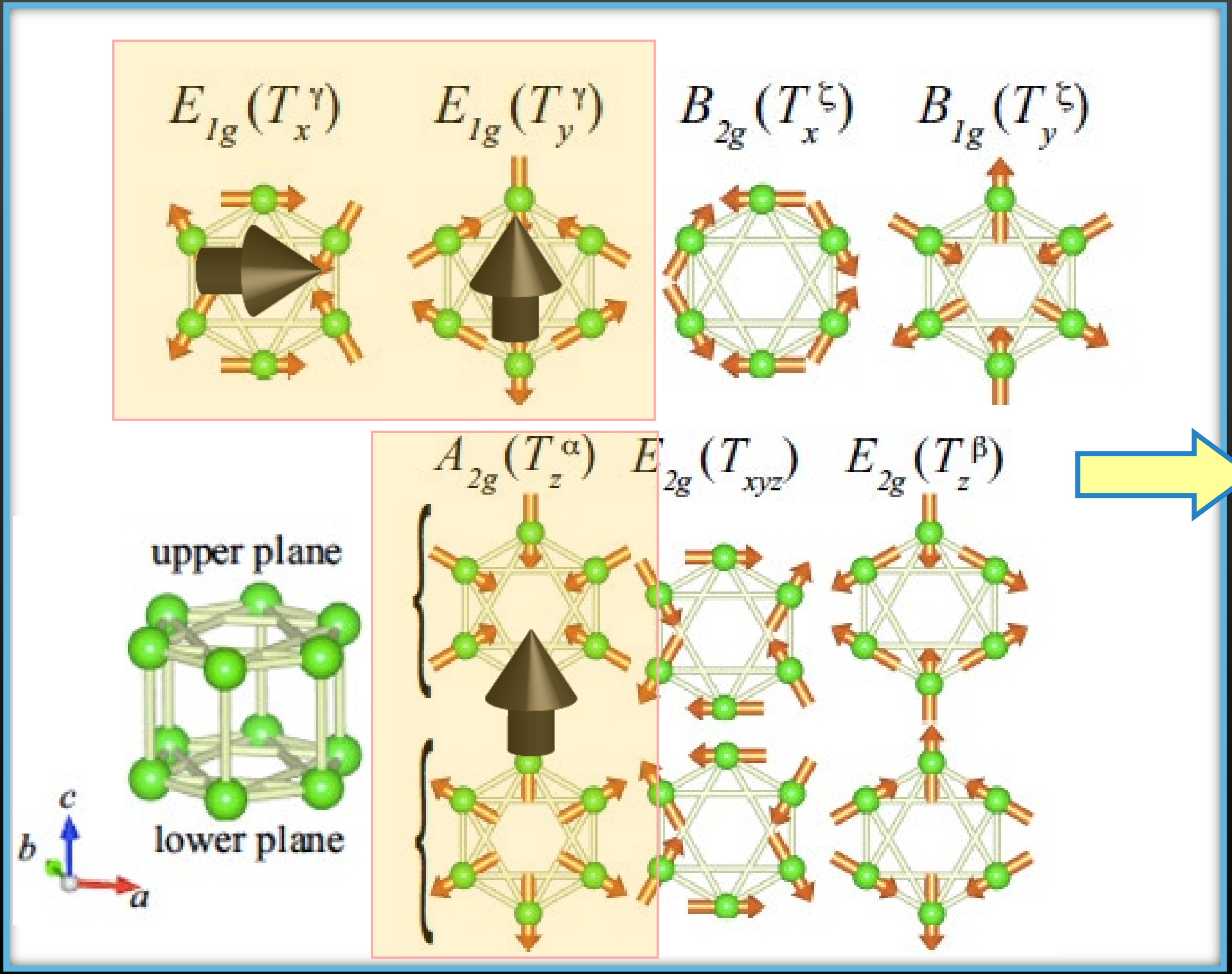


S. N., N. Kiyohara, T. Higo, *Nature* (2015)

$\rho_H = R_0 B + R_S \mu_0 M \sim 0.01\ \mu\Omega\text{cm}$ **vs.** $\rho_H = R_0 B + R_S \mu_0 M + \rho_H^{AF} \sim 3\ \mu\Omega\text{cm}$

The large AHE arises from a momentum-space fictitious field (i.e., Berry curvature) instead of the net *M*

Cluster octupoles in AFM Mn_3Sn

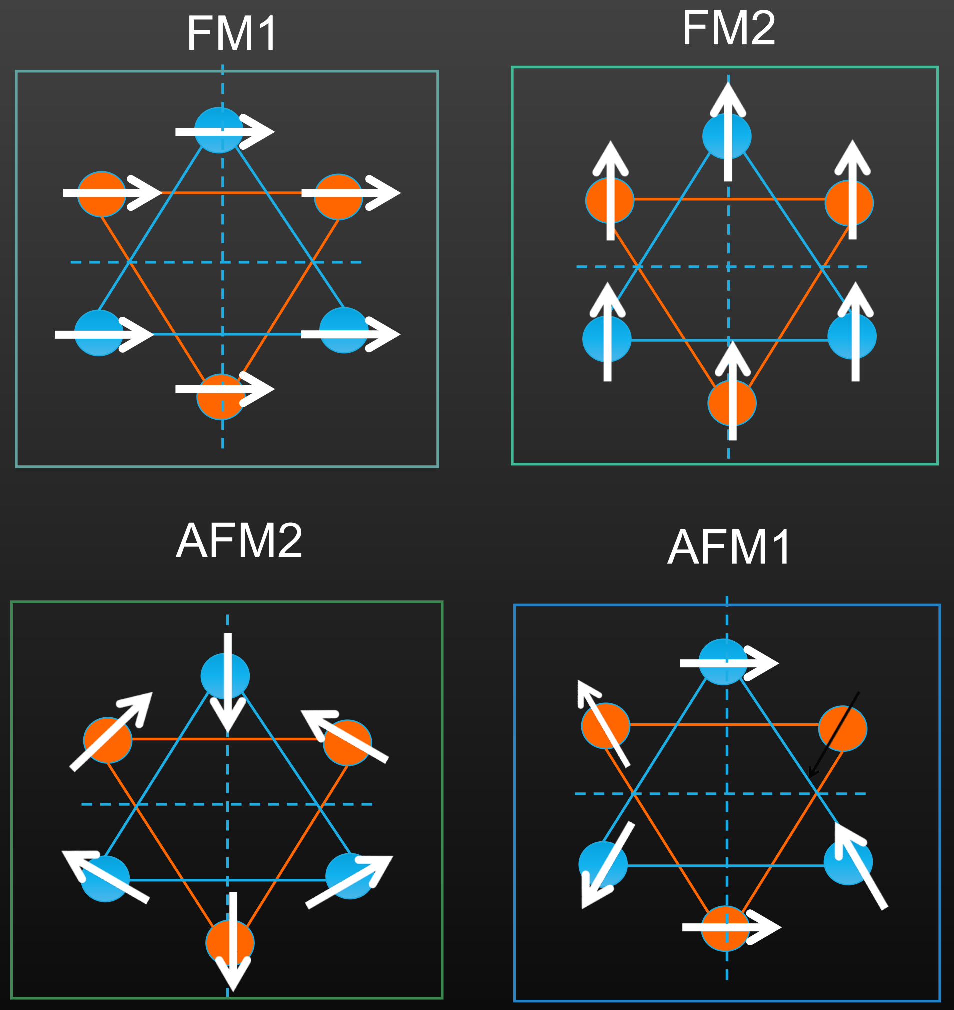


These cluster octupoles behave like a magnetic dipole under time reversal, mirror reflection, and spatial inversion operations

Octupolar polarization plays the same role as *M* in FMs.

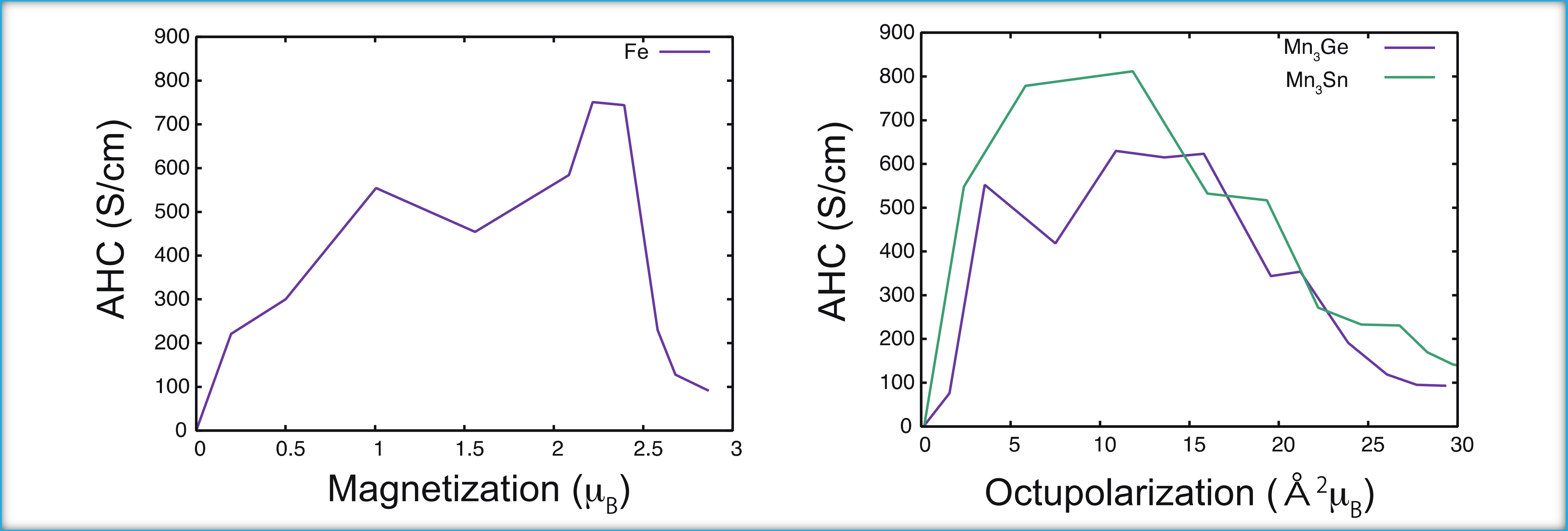
Cluster octupoles in AFM Mn_3Sn

IREP	CMP
A_{2g}	$J_z \equiv M_{10}$
E_{1g}	$J_x \equiv \frac{1}{\sqrt{2}}(-M_{11} + M_{1-1})$
	$J_y \equiv \frac{i}{\sqrt{2}}(M_{11} + M_{1-1})$
A_{1u}	$Q_{3z^2-r^2} \equiv M_{20}$
E_{2u}	$Q_{x^2-y^2} \equiv \frac{1}{\sqrt{2}}(M_{22} + M_{2-2})$
	$Q_{xy} \equiv \frac{i}{\sqrt{2}}(-M_{22} + M_{2-2})$
E_{1u}	$Q_{zx} \equiv \frac{1}{\sqrt{2}}(-M_{21} + M_{2-1})$
	$Q_{yz} \equiv \frac{i}{\sqrt{2}}(M_{21} + M_{2-1})$
A_{2g}	$T_z^\alpha \equiv M_{30}$
E_{1g}	$T_x^\gamma \equiv \frac{1}{\sqrt{2}}(-M_{31} + M_{3-1})$
	$T_y^\gamma \equiv \frac{i}{\sqrt{2}}(M_{31} + M_{3-1})$
E_{2g}	$T_{xyz} \equiv \frac{i}{\sqrt{2}}(-M_{32} + M_{3-2})$
	$T_z^\beta \equiv \frac{1}{\sqrt{2}}(M_{32} + M_{3-2})$
B_{2g}	$T_x^\zeta \equiv \frac{1}{\sqrt{2}}(-M_{33} + M_{3-3})$
B_{1g}	$T_y^\zeta \equiv -\frac{i}{\sqrt{2}}(M_{33} + M_{3-3})$



AF structure in Mn_3Sn = ferroic order of octupoles with the E_{1g} representation

Anomalous Hall effect in Mn_3Sn within the framework of CMP

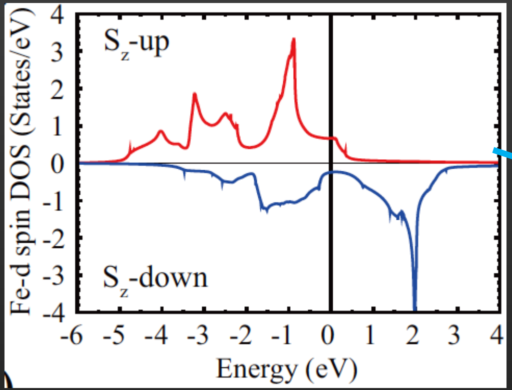


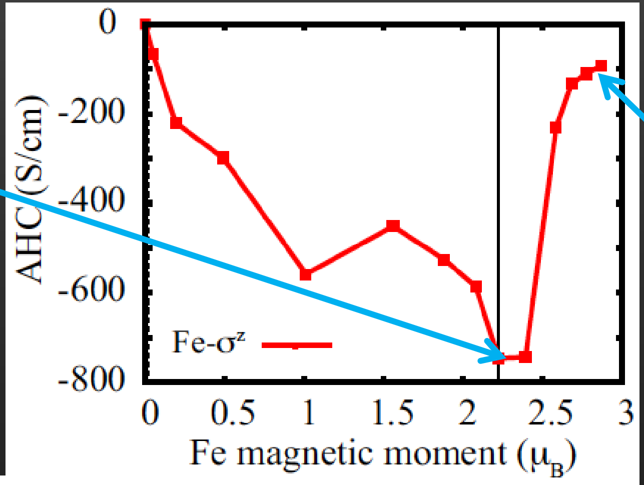
The cluster multipole theory allows us to discuss anomalous Hall conductivity (AHC) in FM and AFM with a unified framework

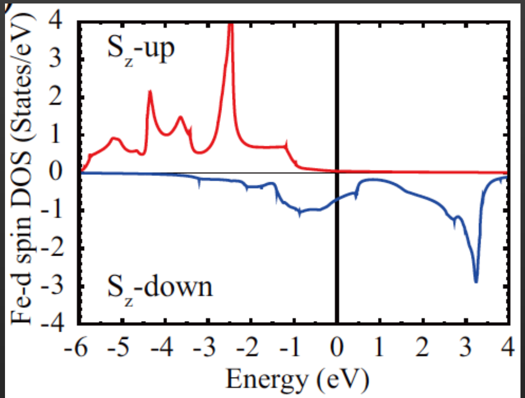
Cluster multipoles can effectively characterize the magnetic and transport properties of AFM

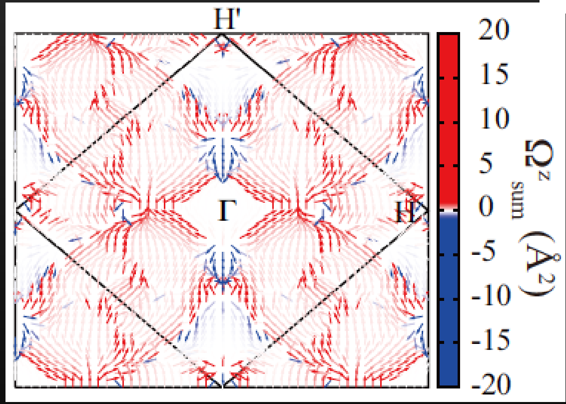
AHE and spin splitting of bcc-Fe

FM states of bcc-Fe



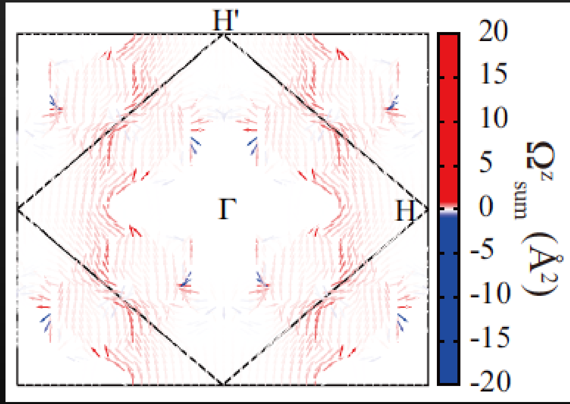






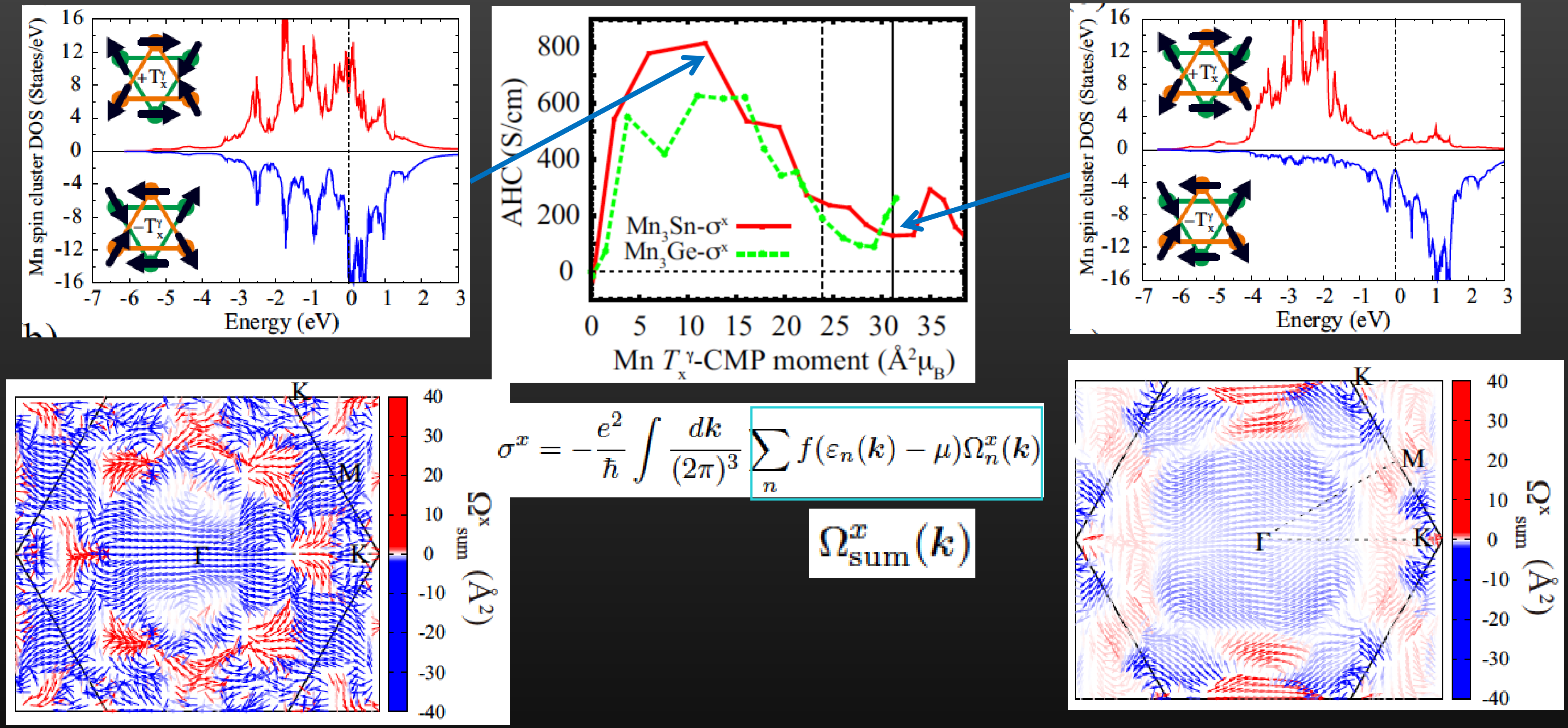
$$\sigma^z = -\frac{e^2}{\hbar}\int \frac{dk}{(2\pi)^3} \sum_n f(\varepsilon_n(\boldsymbol{k}) - \mu)\Omega_n^z(\boldsymbol{k})$$

$$\Omega^z_{\text{sum}}(\boldsymbol{k})$$



AHE and CMP orbital splitting of Mn_3Sn

AFM states of Mn_3Z (Z=Sn, Ge)

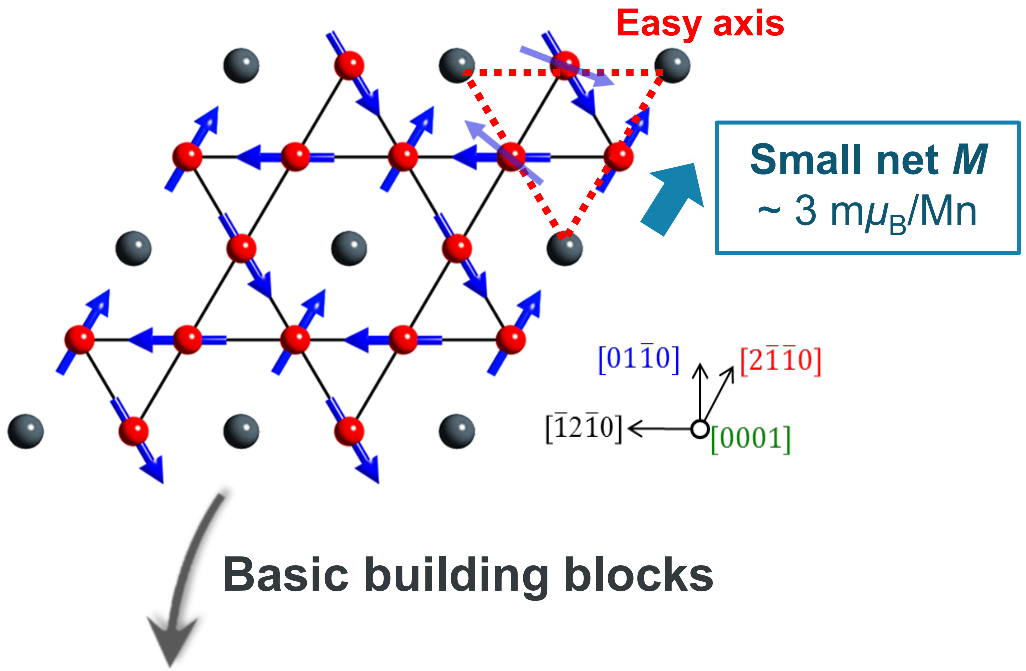


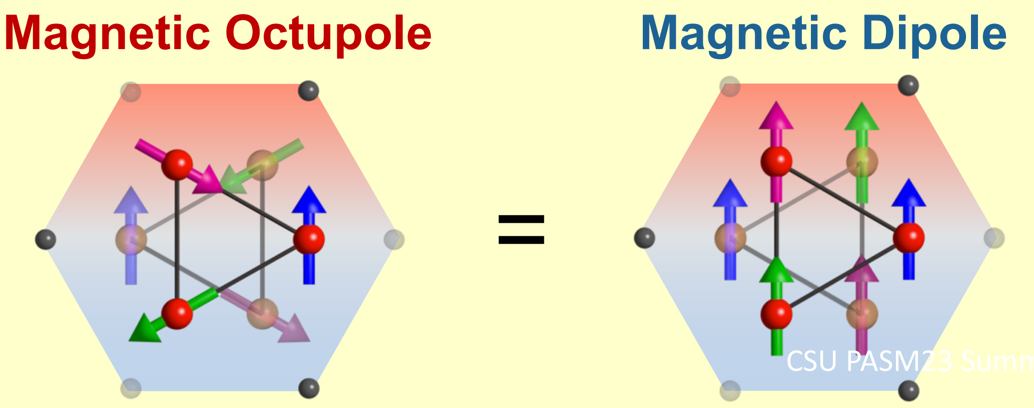
$$\sigma^x = -\frac{e^2}{\hbar}\int \frac{dk}{(2\pi)^3} \sum_n f(\varepsilon_n(\boldsymbol{k}) - \mu)\Omega_n^x(\boldsymbol{k})$$

$$\Omega^x_{\rm sum}(\boldsymbol{k})$$

Summary: Cluster octupole ordering in Mn_3Sn

Noncollinear AFM order at T_N = 430 K





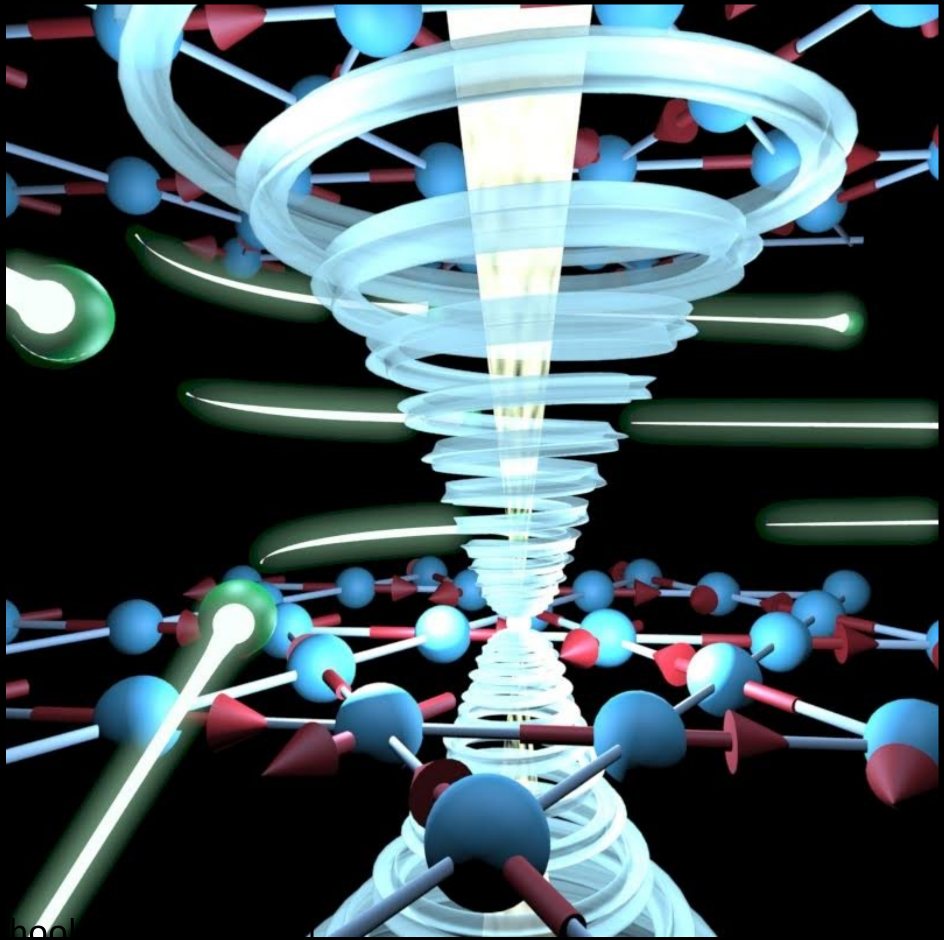
A group of six spins forms **a cluster octupole moment**, which is highly tunable by a magnetic field, electrical current, and strain.

The AFM order in Mn_3Sn is **a ferroic order of cluster octupoles**, which macroscopically breaks time-reversal symmetry.

Cluster octupole polarization *K*~ Berry Curvature (the momentum-space fictitious magnetic field)

$$\sigma^H_{\alpha\beta} = \frac{e^2}{2\pi h}\epsilon_{\alpha\beta\gamma}K_\gamma$$

Strain Control of AHE in Mn_3Sn

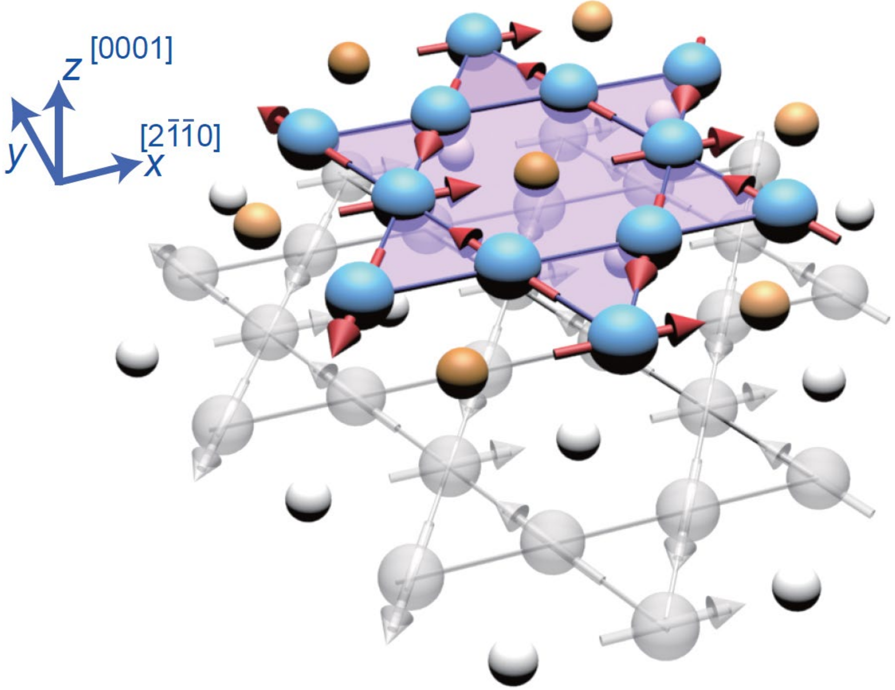


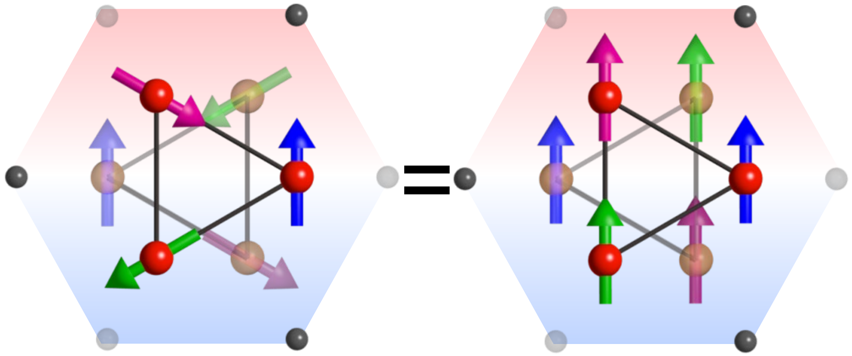
Nature Physics **volume 18**, 1086–1093 (2022)

Magnetic Multipole

Suzuki, Arita et al., PRB 094406(2017).

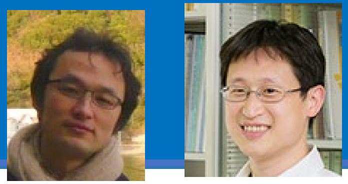
NonCollinear AFM T_N = 430 K





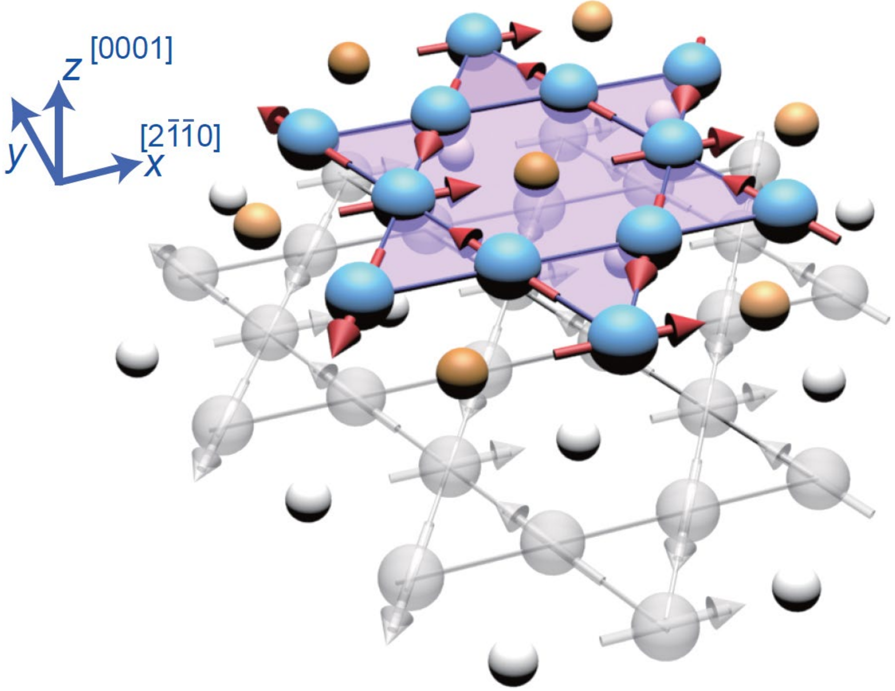
The Same Mag. Space Group
Breaking Time Reversal Symm.

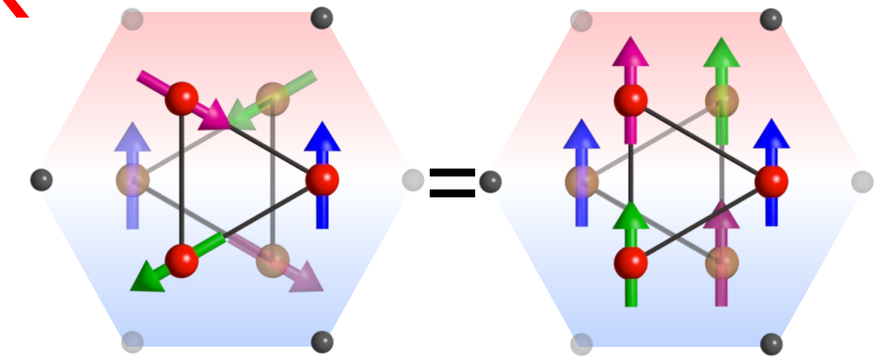
Magnetic Octupole



Suzuki, Arita et al., PRB 094406(2017).

NonCollinear AFM T_N = 430 K





Magnetic Octupole (Rank = 3)

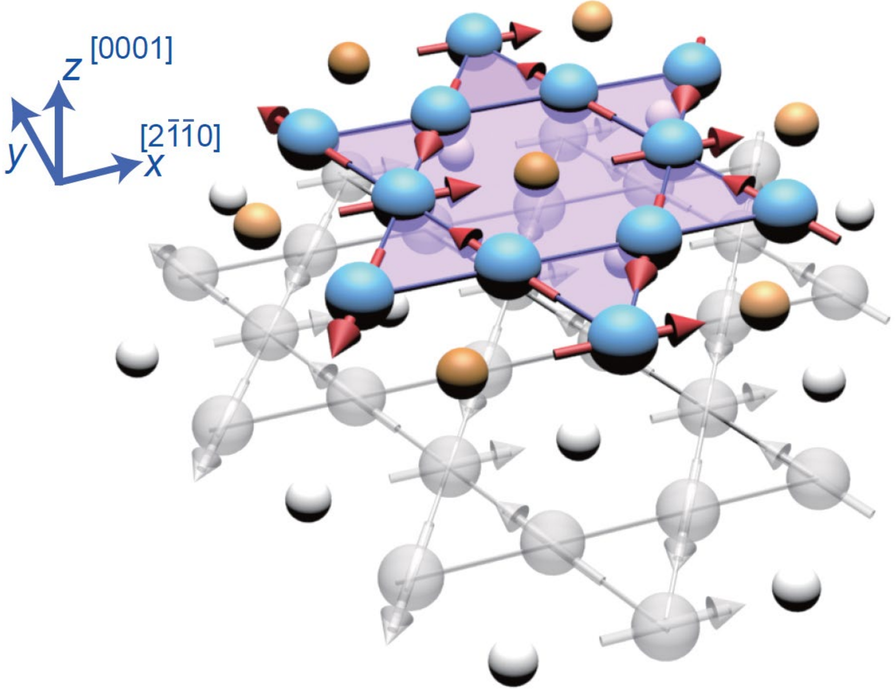
Dipole (Rank =1)

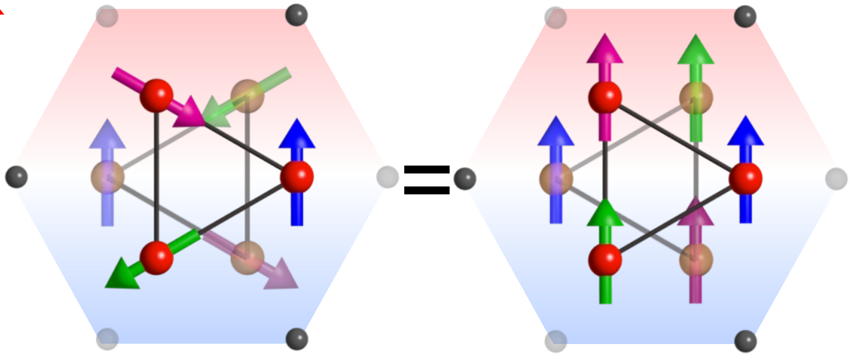
The Same Mag. Space Group
Breaking Time Reversal Symm.

Magnetic Octupole

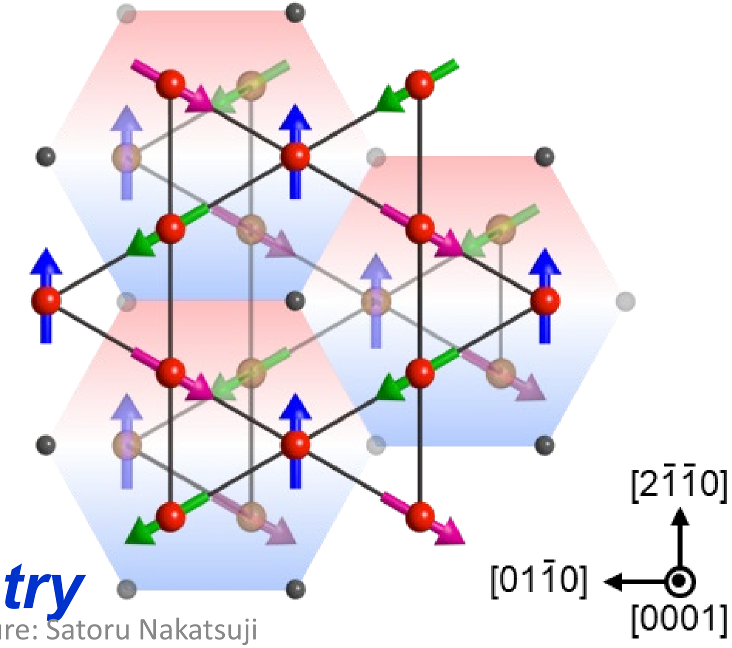

Suzuki, Arita et al., PRB 094406(2017).

NonCollinear AFM T_N = 430 K





Magnetic Octupole FM



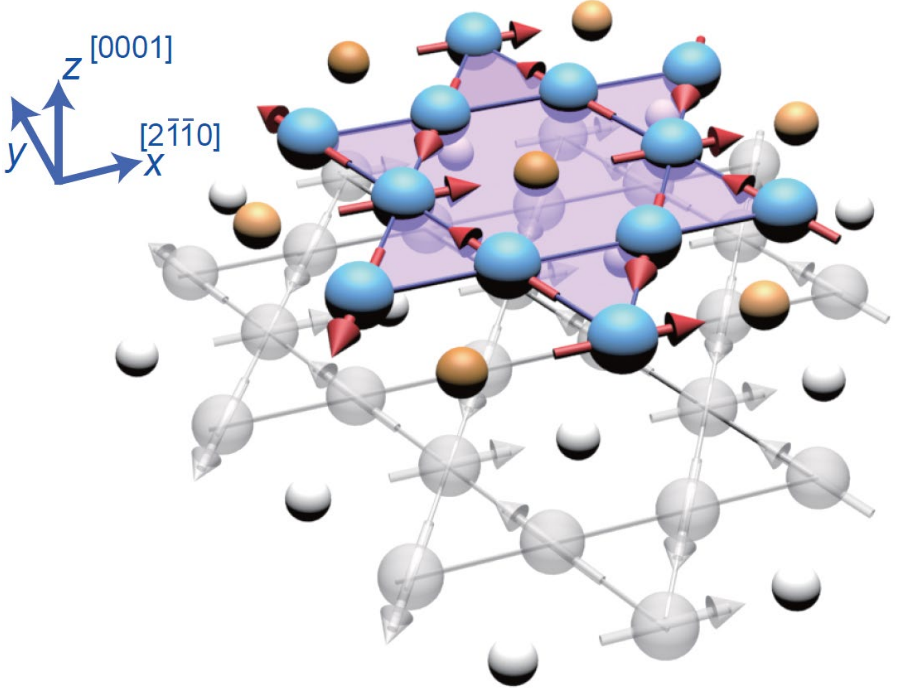
Ferroic Order of Magnetic Octupole

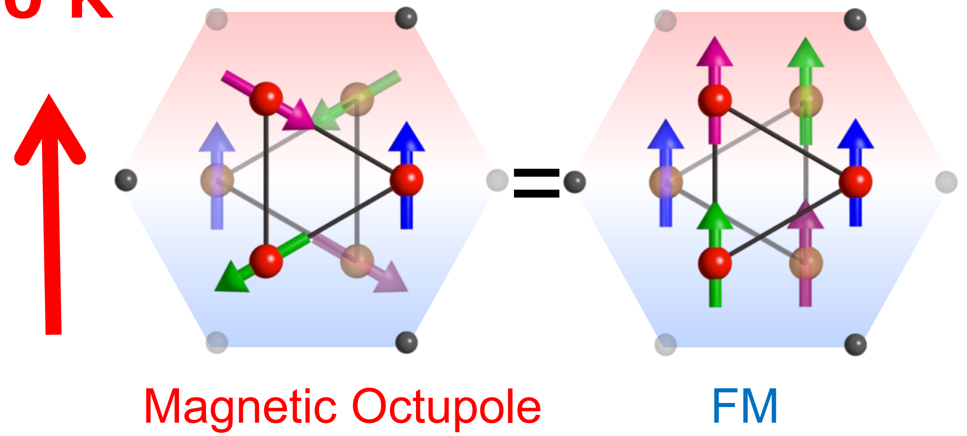
Breaking Time Reversal Symmetry

Magnetic Octupole

Suzuki, Arita et al., PRB 094406(2017).

NonCollinear AFM T_N = 430 K





K: octupole polarization

~ Berry Curvature

Ferroic Order of Magnetic Octupole

$$\sigma^H_{\alpha\beta} = \frac{e^2}{2\pi h}\epsilon_{\alpha\beta\gamma}K_\gamma$$

Breaking Time Reversal Symmetry

Piezomagnetic effect in antiferromagnets

- For certain types of antiferromagnets, strain breaks the symmetry between magnetic sublattices, and induces net magnetization linear in the applied strain

- Piezomagnetic effect :

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} & \Lambda_{26} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & \Lambda_{36} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$$

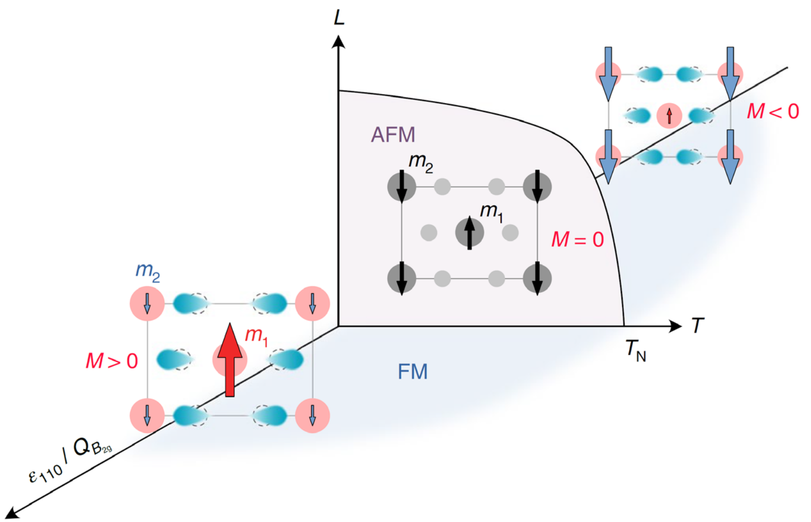
- **Λ** is non-zero for AFM that macroscopically break time reversal symmetry or preserve time reversal symmetry *only* in combination with rotation and reflection. 66 out of 122 magnetic point groups allow piezomagnetic effect

E. Dzyaloshinskii, JETP 33, 807 (1957) B. A. Tavger and V. M. Zaitzev, *J. Exp. Theor. Phys.* **3** (1956)

Piezomagnetic effect in collinear antiferromagnet CoF_2 and MnF_2

- Tetragonal structure, 2 distinct transition metal sites
- Strain along [110] direction, breaks the symmetry of the sublattice moments → ferrimagnetic moment along [001]

Moriya, T., *Journal of Physics and Chemistry of Solids* **11** (1959)



S. A. Disa, *et al. Nature Physics* **16** (2020)

- In the presence of piezomagnetic effect, under a constant field, a static stress can mediate 180° AF domain reversal

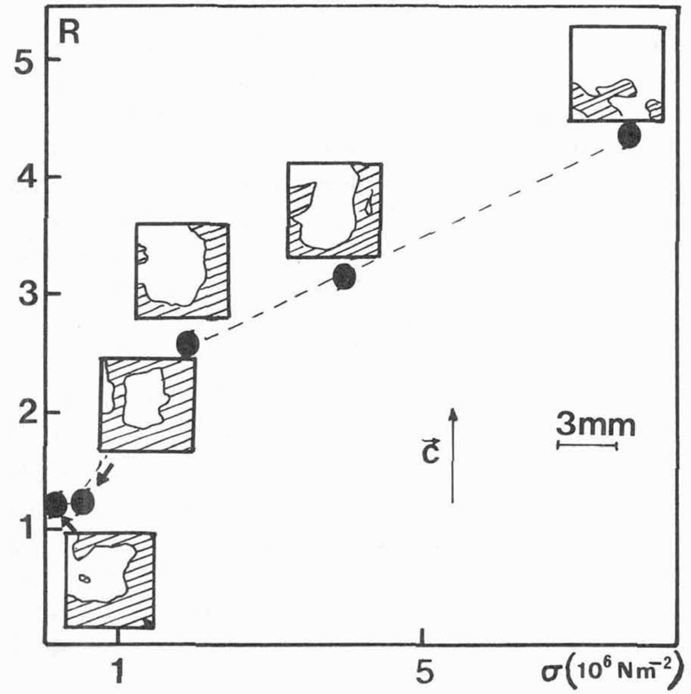


Fig. 1. — Flipping ratio R as a function of the stress applied during the cooling of the MnF_2 crystal $(4 \times 4 \times 2.5\ mm^3)$ through T_N in a 0.01 T magnetic field, and schematic drawing of the topographs recorded after removing the field, at 20 K, with neutrons polarized along [001].

Baruchel, J., *et al. Le Journal de Physique Colloques* **49** (1988)

Has been seen mostly in AF insulators

Piezomagnetic effect in Weyl semimetal Mn_3Sn

- The magnetic structure of 120º antichiral phase macroscopically breaks time-reversal symmetry, piezomagnetic effects are allowed

- Its magnetic point group symmetry (*m'm'm'*) dictates:

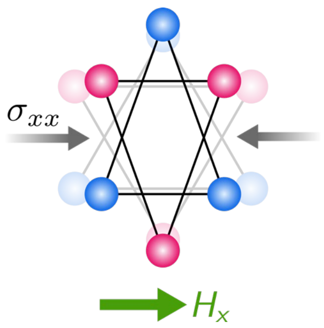
$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & 0 & 0 & \Lambda_{16} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & 0 & 0 & \Lambda_{26} \\ 0 & 0 & 0 & \Lambda_{34} & \Lambda_{35} & 0 \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$$

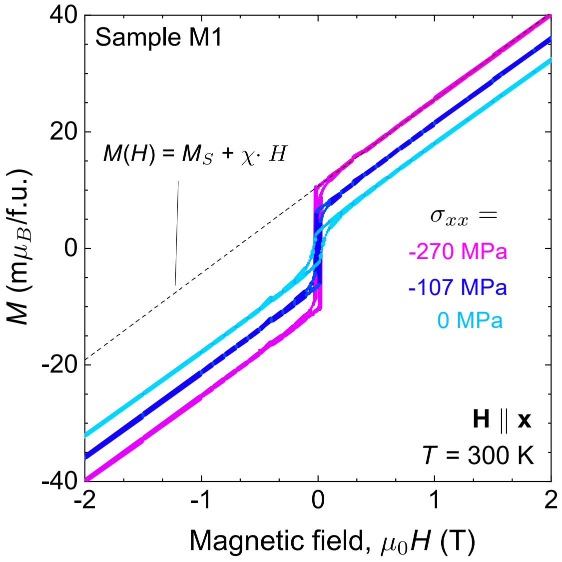
Source: Bilbao crystallographic server

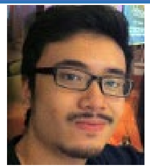
- In-plane stress couples to magnetization,

for example $M_x = \Lambda_{11}\sigma_{xx} + \Lambda_{12}\sigma_{yy} + \Lambda_{16}\sigma_{xy}$

Magnetization of Mn_3Sn under in-plane uniaxial compression







- We fit the data with:

$$M(H) = M_S + \chi H$$

Spontaneous compontent

Field-induced compontent

→ M_S is enhanced by in-plane stress

→ χ is insensitive to stress

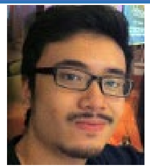
Ikhlas, Dasgupta, et al., *Nature Physics* volume 18, pages 1086–1093 (2022)

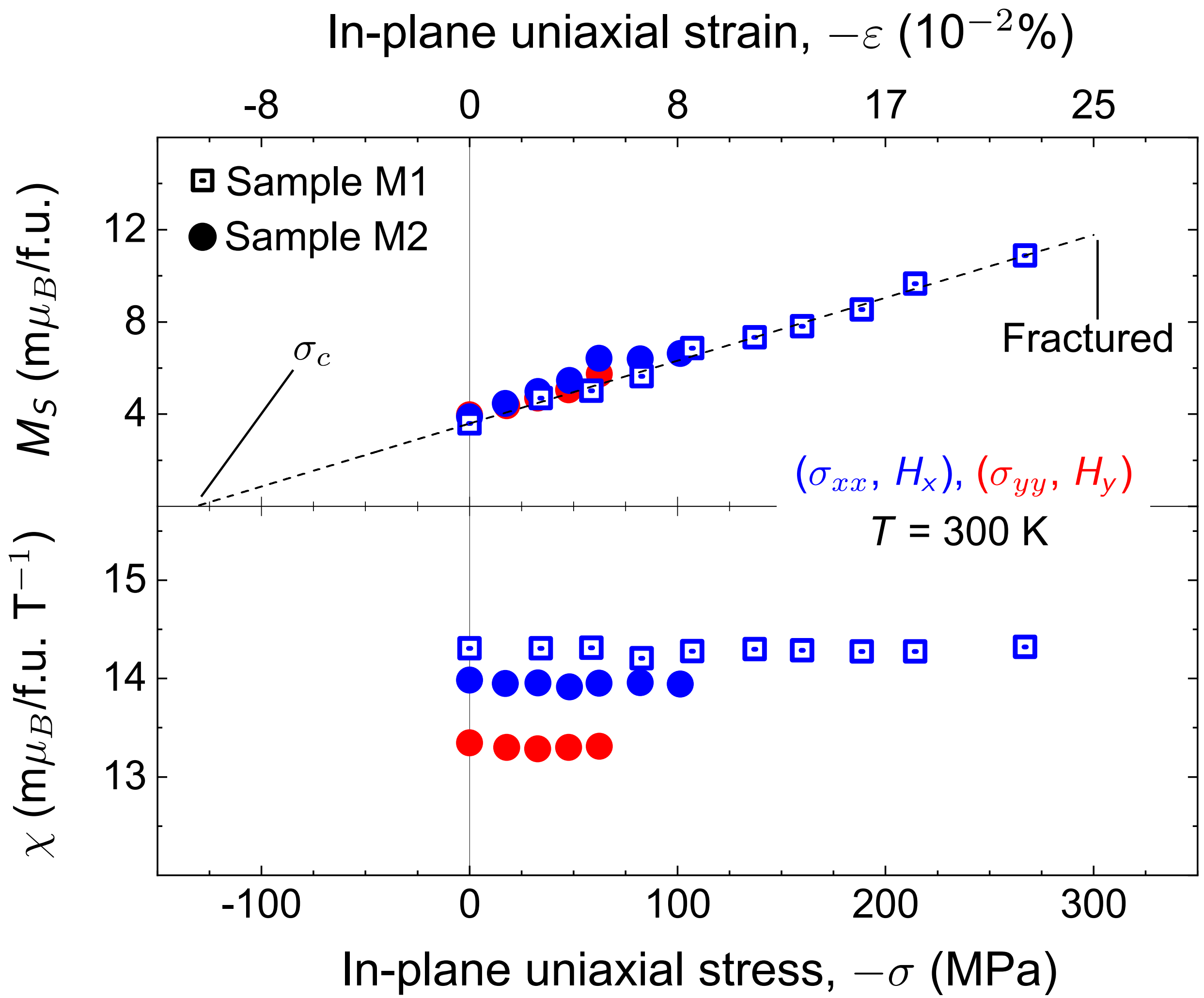
Stress-dependence of spontaneous magnetization M_S

$M_S = M_S(\sigma = 0) + \Lambda_{11}\sigma_{xx}$

→ Evidence for piezomagnetism

❑ $|\Lambda_{11}|$ ~ 0.027 mμ_B/f.u. GPa^{-1} (0.078 Gauss/MPa)



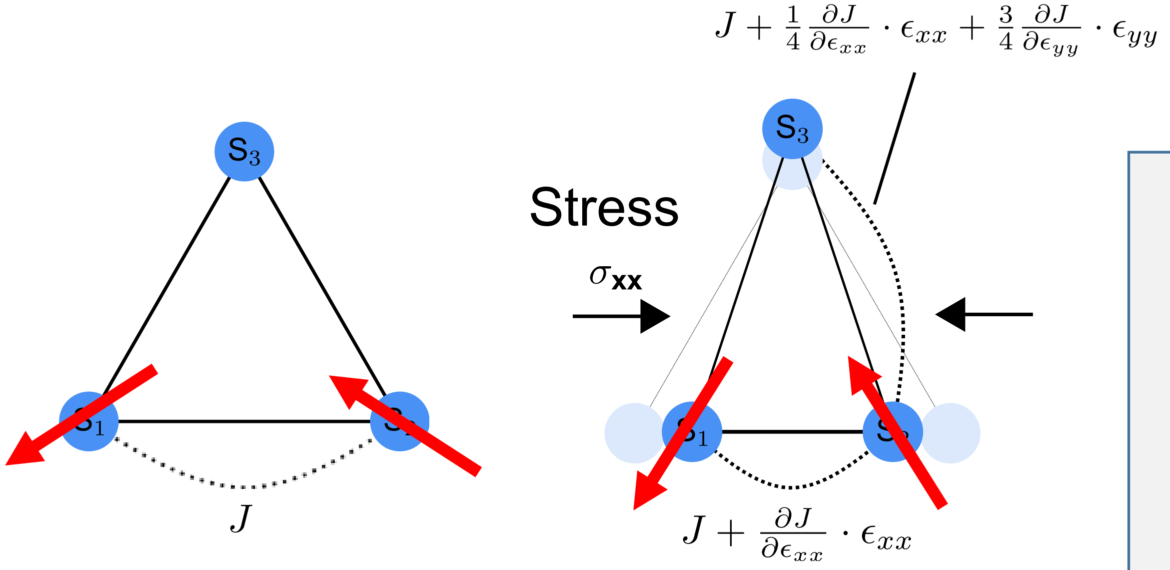


Young's Modulus of Mn_3Sn = 121 GPa

Nature Physics **volume 18**, 1086–1093 (2022)

Microscopic origin of piezomagnetic effect in Mn_3Sn

- Exchange interaction *J* depends on the distance between magnetic ions (*exchange striction*)



$$H_{ex} = J\,(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_1)$$

$$H'_{ex} = \sum_{i \neq j} J_{ij} \left(\mathbf{S}_i \cdot \mathbf{S}_j\right)$$

$$= H_{ex} + \Delta H_{ex}$$

ΔH_{ex} = Correction due to exchange striction

Illustration of piezomagnetic effect in Mn_3Sn

- strain-dependent magnetization

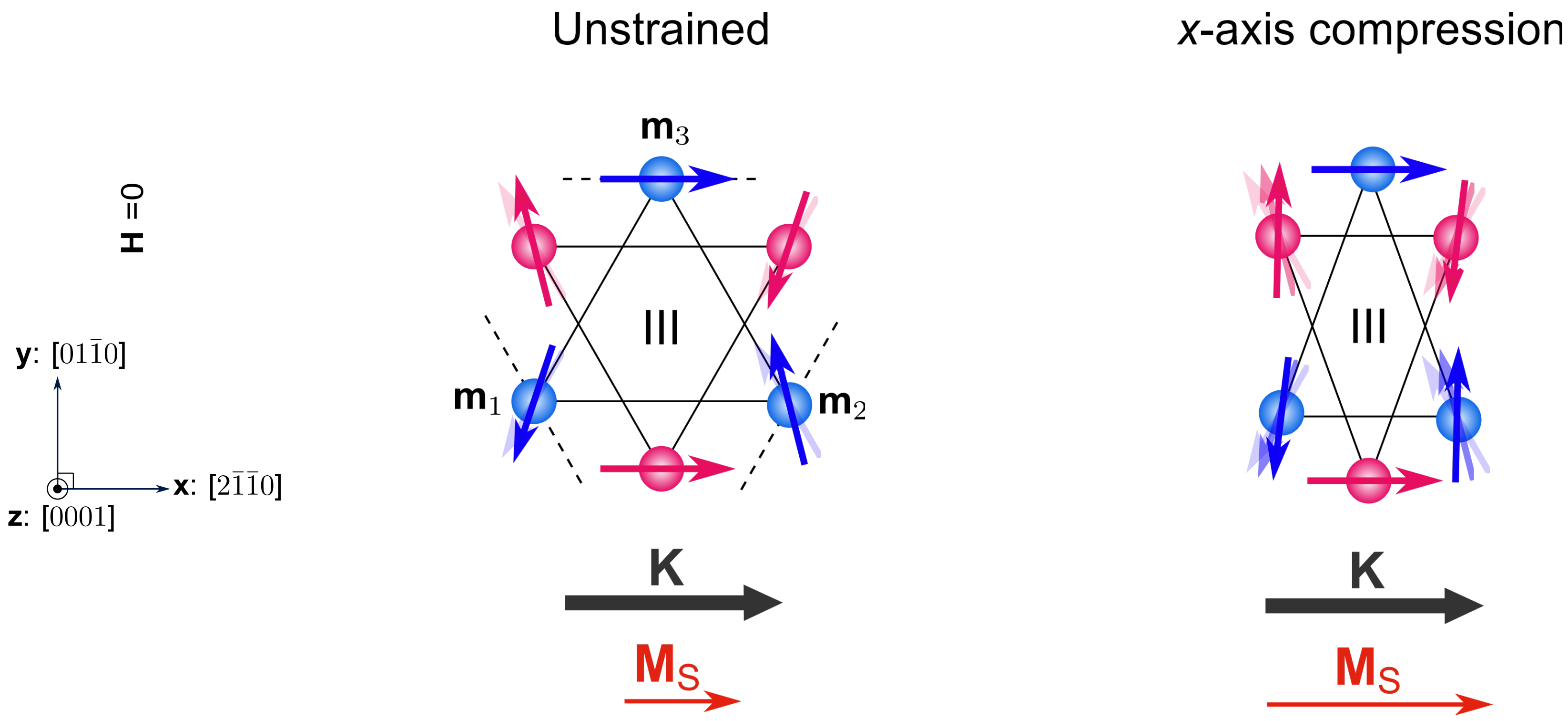
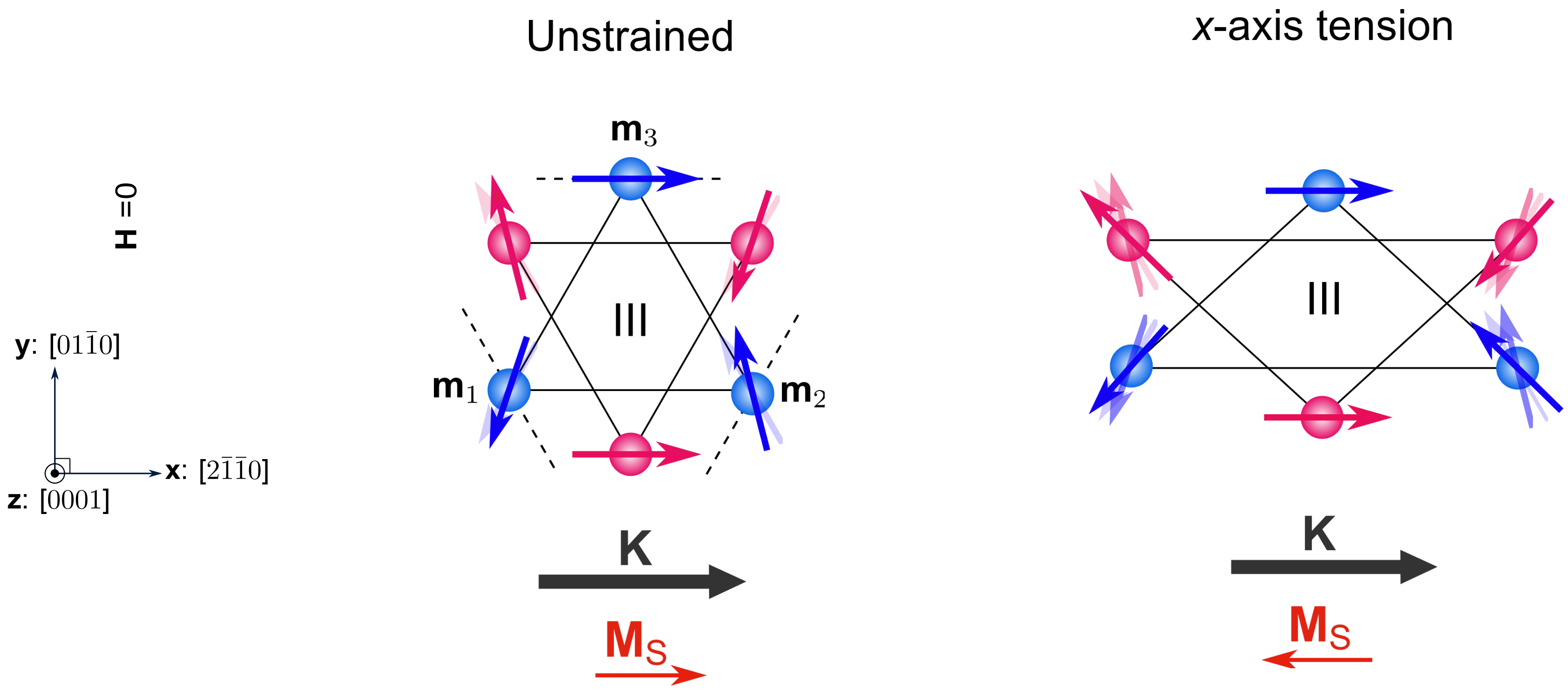


Illustration of piezomagnetic effect in Mn_3Sn

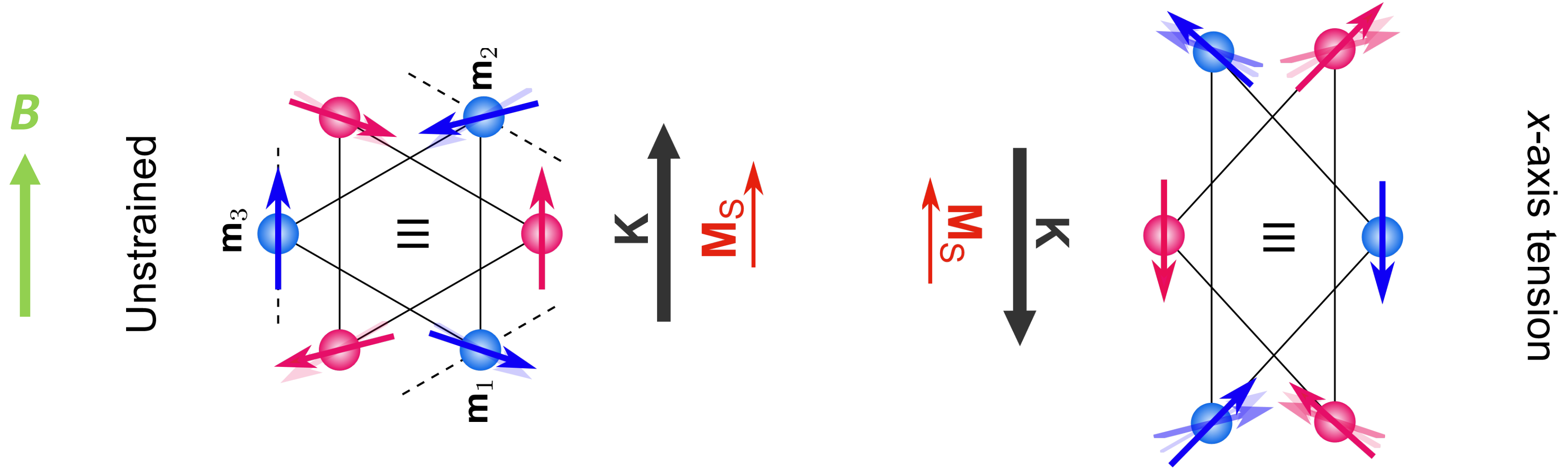
❑strain-dependent magnetization



❑In-plane tension may rotate **M** to the opposite direction to **K**

Illustration of piezomagnetic effect in Mn_3Sn

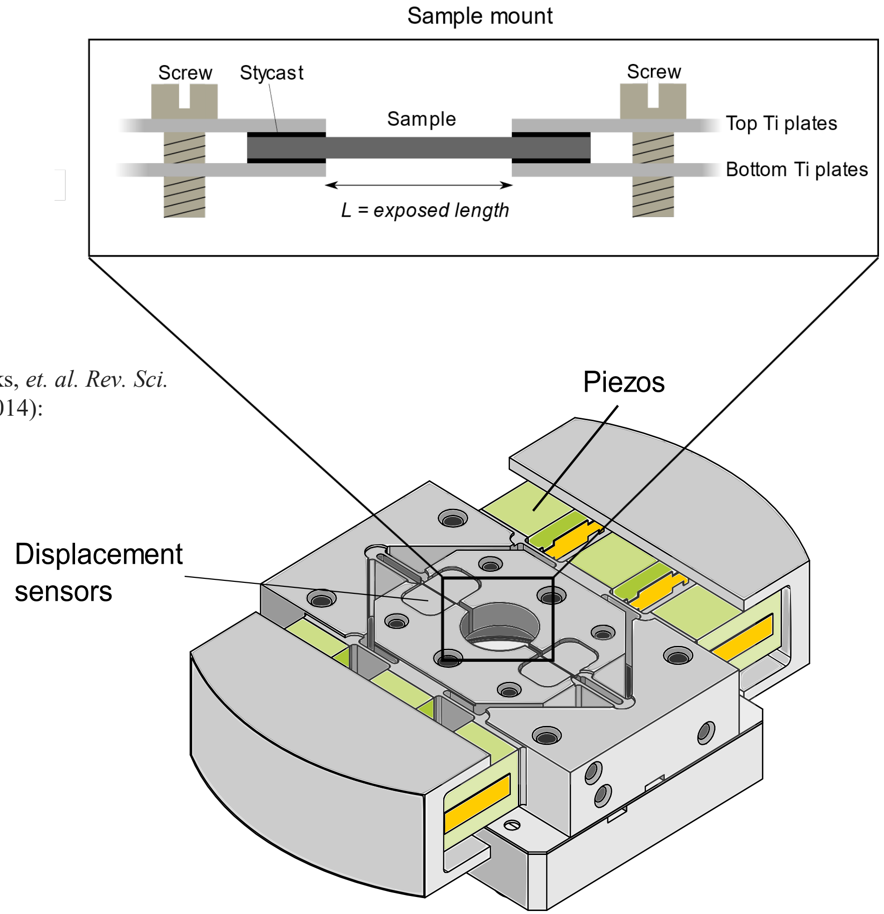
❑strain-dependent magnetization



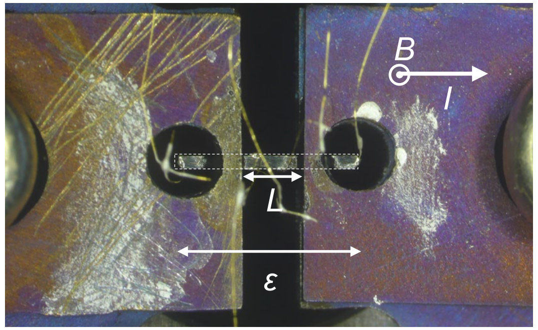
❑In-plane tension may rotate **M** to the opposite direction to **K**

→ **Leads to a sign change in the anomalous Hall effect**

Uniaxial strain cell and sample mounting



C.W. Hicks, *et. al. Rev. Sci. Ins.* **85** (2014):



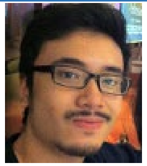
Strain: $\varepsilon = {}^{\Delta L}/_{L}$ $\quad \Delta L = \varepsilon_0 A \left({}^{1}/_{C} - {}^{1}/_{C_0} \right)$

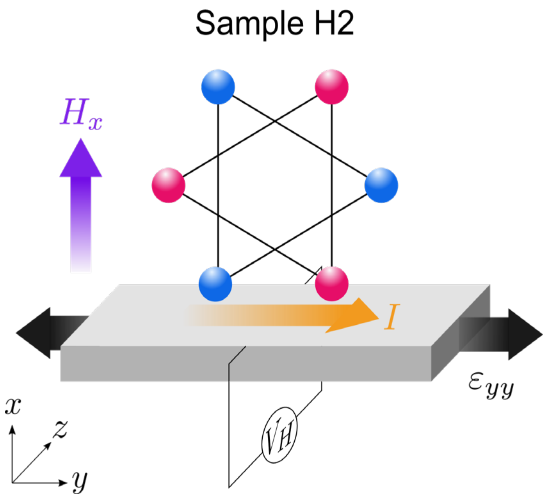
A = area of parallel plate capacitor

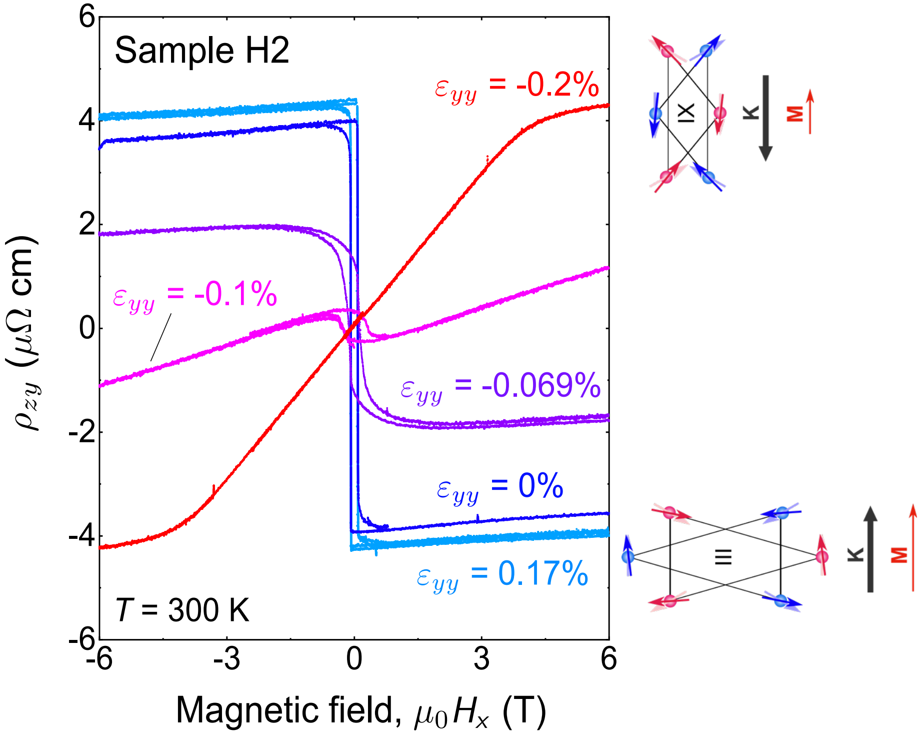
C = capacitance of displacement sensor (pF)

C_0 = initial capacitance of displacement sensor (pF)

Anomalous Hall effect under in-plane uniaxial strain



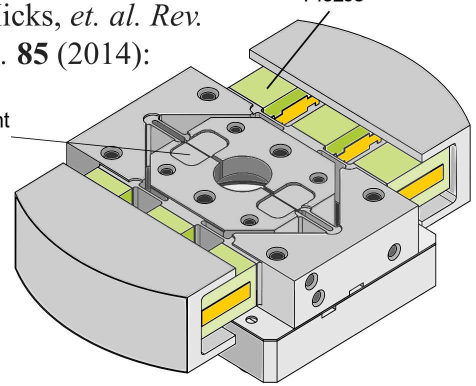


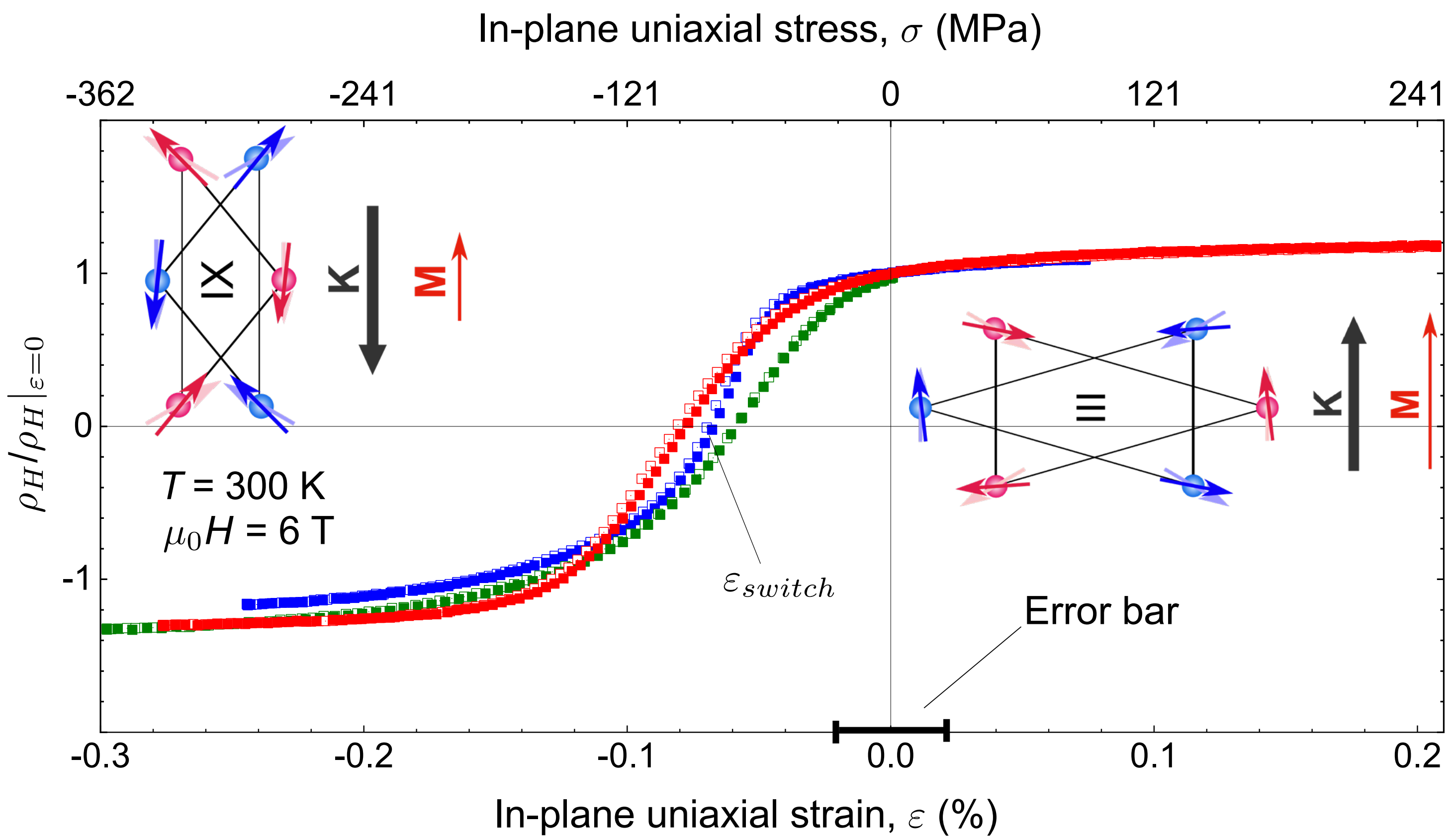


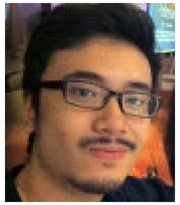
- ❑ AHE couples to in-plane uniaxial strain → sign change of AHE under compressive strain

Strain dependence of normalized anomalous Hall resistivity

C.W. Hicks, *et. al. Rev. Sci. Ins.* **85** (2014):







- ❑ Hall resistivity can change sign while the sign of the magnetization remains the same

→ Evidence that the AHE in Mn_3Sn is controlled by the octupolar order, and **not** the dipolar magnetization

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